UNIT I

OSCILLATION, ULTRASONICS AND DIELECTRIC MATERIALS

1.1 INTRODUCTION TO OSCILLATIONS

- A wave is a disturbance which travels through available space or medium. In a wave motion, energy is transferred from one place to other place due to the repeated periodic motion of the medium particles.
- When a wave passes the medium particles vibrate or oscillate about their mean position. The oscillating particles perform a periodic motion called **Harmonic Motion**.
- The three features associated with a **Low Energy Waves** are (i) energy is transmitted, (ii) medium is not transmitted and (iii) return to equilibrium is involved.
- Although the medium particles vibrate about their mean position, they transfer energy by transferring motion from one particle to the another at a regular interval of time.
- For propagation of a wave the medium must satisfy the following conditions :
    - ➢ The medium must be elastic, so that it returns to its original position after oscillation.
    - ➢ The medium particles must have inertia so that it swings to the other side of the oscillation.
    - ➢ The medium must be viscous so that the energy exchange takes place between the medium particles.
- The understanding of wave is an important aspect as it transfers the energy from one place to the other place. The most common types of waves are light, sound and heat etc. As wave propagation involves the oscillations of medium particles, we must understand the oscillations to understand the concept of wave propagation.

1.2 FREE OSCILLATION [Dec. 18]

- The motion of pendulum, piston of an engine, earth around the sun follows the same path and repeats it after equal intervals of time. Such type of motion is called as **Periodic Motion**. The periodic motion can be either circular or linear depending upon whether it moves along circular path or linear path.
- When we give a push to a pendulum or a string fixed at two ends is plucked and left free to oscillate, it oscillates with a frequency given by,

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{g}{l}\right)} \quad \text{... for pendulum ...} \quad (1.1)$$

$$f = \frac{1}{2l}\sqrt{\left(\frac{T}{m}\right)} \quad \text{... for string ...} \quad (1.2)$$

- This frequency is called **Natural Frequency** of the vibrating system. The frequency with which a body vibrates freely at its own is called its **Natural Frequency**.
- If no resistance is offered to the movement of the vibrating body, the body will keep on vibrating indefinitely. Such a vibration is called **Free Oscillation** or **Vibration**.
- In a free oscillation or vibration, whenever a body is disturbed, it vibrates with its own natural frequency for infinite time. But in practice, frictionless system is not possible and amplitude of vibrating body decreases slowly to zero. When the friction is very less, the system can be considered as **Free Oscillation**.
- Consider motion of a particle of mass m on which a restoring force is acting, such that the particle performs harmonic oscillation. For harmonic oscillations, the restoring force is linearly proportional to the displacement i.e. $-ky$ where k is the restoring force constant and negative sign indicates that it acts in opposite direction to displacement.

i.e. $$f = -ky \quad \text{...} \quad (1.3)$$

According to Newton's law

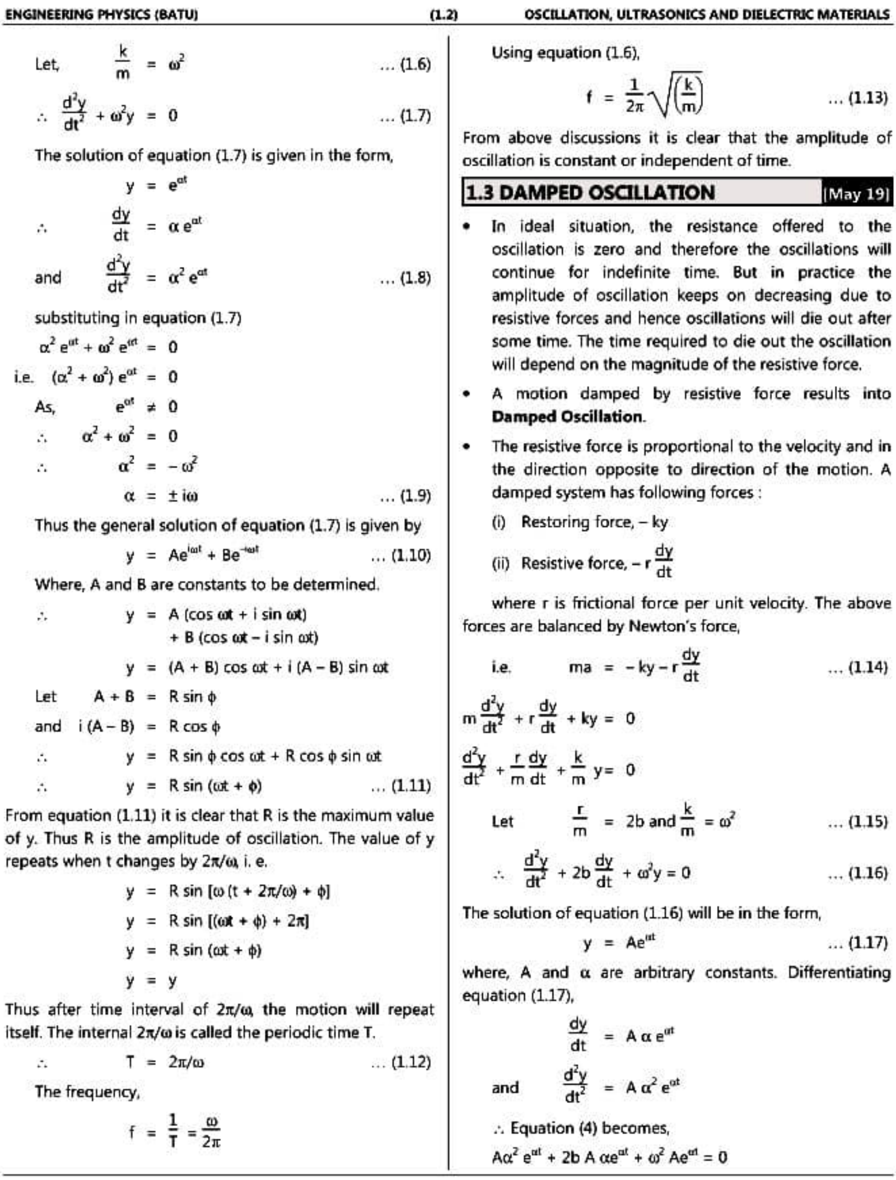
$$f = ma \quad \text{...} \quad (1.4)$$

i.e. $$f = m\frac{d^2y}{dt^2} \quad \text{...} \quad (1.5)$$

Comparing equations (1.3) and (1.5);

$$\frac{md^2y}{dt^2} = -ky$$

i.e. $$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$



Let, $$\frac{k}{m} = \omega^2 \qquad \dots (1.6)$$

$$\therefore \frac{d^2y}{dt^2} + \omega^2 y = 0 \qquad \dots (1.7)$$

The solution of equation (1.7) is given in the form,

$$y = e^{\alpha t}$$

$$\therefore \frac{dy}{dt} = \alpha e^{\alpha t}$$

and $$\frac{d^2y}{dt^2} = \alpha^2 e^{\alpha t} \qquad \dots (1.8)$$

substituting in equation (1.7)

$$\alpha^2 e^{\alpha t} + \omega^2 e^{\alpha t} = 0$$

i.e. $$(\alpha^2 + \omega^2) e^{\alpha t} = 0$$

As, $$e^{\alpha t} \neq 0$$

$$\therefore \alpha^2 + \omega^2 = 0$$

$$\therefore \alpha^2 = -\omega^2$$

$$\alpha = \pm i\omega \qquad \dots (1.9)$$

Thus the general solution of equation (1.7) is given by

$$y = Ae^{i\omega t} + Be^{-i\omega t} \qquad \dots (1.10)$$

Where, A and B are constants to be determined.

$$\therefore y = A(\cos \omega t + i \sin \omega t) + B(\cos \omega t - i \sin \omega t)$$

$$y = (A + B) \cos \omega t + i(A - B) \sin \omega t$$

Let $$A + B = R \sin \phi$$

and $$i(A - B) = R \cos \phi$$

$$\therefore y = R \sin \phi \cos \omega t + R \cos \phi \sin \omega t$$

$$\therefore y = R \sin (\omega t + \phi) \qquad \dots (1.11)$$

From equation (1.11) it is clear that R is the maximum value of y. Thus R is the amplitude of oscillation. The value of y repeats when t changes by 2π/ω, i. e.

$$y = R \sin [\omega (t + 2\pi/\omega) + \phi]$$

$$y = R \sin [(\omega t + \phi) + 2\pi]$$

$$y = R \sin (\omega t + \phi)$$

$$y = y$$

Thus after time interval of 2π/ω, the motion will repeat itself. The internal 2π/ω is called the periodic time T.

$$\therefore T = 2\pi/\omega \qquad \dots (1.12)$$

The frequency,

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Using equation (1.6),

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k}{m}\right)} \qquad \dots (1.13)$$

From above discussions it is clear that the amplitude of oscillation is constant or independent of time.

1.3 DAMPED OSCILLATION

[May 19]

- In ideal situation, the resistance offered to the oscillation is zero and therefore the oscillations will continue for indefinite time. But in practice the amplitude of oscillation keeps on decreasing due to resistive forces and hence oscillations will die out after some time. The time required to die out the oscillation will depend on the magnitude of the resistive force.
- A motion damped by resistive force results into **Damped Oscillation.**
- The resistive force is proportional to the velocity and in the direction opposite to direction of the motion. A damped system has following forces :

(i) Restoring force, – ky

(ii) Resistive force, $-r\frac{dy}{dt}$

where r is frictional force per unit velocity. The above forces are balanced by Newton's force,

i.e. $$ma = -ky - r\frac{dy}{dt} \qquad \dots (1.14)$$

$$m\frac{d^2y}{dt^2} + r\frac{dy}{dt} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

Let $$\frac{r}{m} = 2b \text{ and } \frac{k}{m} = \omega^2 \qquad \dots (1.15)$$

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = 0 \qquad \dots (1.16)$$

The solution of equation (1.16) will be in the form,

$$y = Ae^{\alpha t} \qquad \dots (1.17)$$

where, A and α are arbitrary constants. Differentiating equation (1.17),

$$\frac{dy}{dt} = A\alpha e^{\alpha t}$$

and $$\frac{d^2y}{dt^2} = A\alpha^2 e^{\alpha t}$$

∴ Equation (4) becomes,

$$A\alpha^2 e^{\alpha t} + 2b A\alpha e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$$

$$\therefore Ae^{\alpha t}(\alpha^2 + 2b\alpha + \omega^2) = 0$$

$$\text{But,} \quad Ae^{\alpha t} \neq 0$$

$$\therefore \alpha^2 + 2b\alpha + \omega^2 = 0 \quad \ldots (1.18)$$

∴ Roots of equation (1.18) gives

$$\alpha = \frac{-2b \pm \sqrt{(4b^2 - 4\omega^2)}}{2}$$

$$\alpha = -b \pm \sqrt{(b^2 - \omega^2)} \quad \ldots (1.19)$$

The general solution of equation (1.16) is given by,

$$y = A\,e^{(-b + \sqrt{(b^2 - \omega^2)})\,t} + Be^{(-b - \sqrt{(b^2 - \omega^2)})\,t} \quad \ldots (1.20)$$

where A and B are arbitrary constant.

Case I : Over Damped or Dead Beat :

- When $b^2 > \omega^2$, $\sqrt{(b^2 - \omega^2)}$ is real and less than b. In this case power of both the exponents is negative. Thus the displacement y consists of two terms both are decreasing exponentially. This type of motion is called **Over Damped** or **Dead Beat**.
- Example of such oscillation is pendulum in thick oil. Fig. 1.1 shows over damped oscillation.

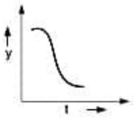


Fig. 1.1 : Over damped oscillation

Case II : Critically Damped :

- When $b^2 = \omega^2$, if we put $b^2 = \omega^2$ in the solution it will not satisfy the differential equation. Therefore, assume that $\sqrt{(b^2 - \omega^2)}$ is not zero but is tending to zero i.e. equal to a very small quantity h.

Therefore solution becomes, from equation (1.20)

$$y = Ae^{(-b+h)t} + Be^{(-b-h)t} \quad \ldots (1.21)$$

$$y = e^{-bt}(Ae^{ht} + Be^{-ht})$$

$$y = e^{-bt}[A(1 + ht + \ldots) + B(1 - ht + \ldots)]$$

$$y = e^{-bt}[(A + B) + ht(A - B) + \ldots]$$

$$y = e^{-bt}[s + ut] \quad \ldots (1.22)$$

$$\text{where,} \quad S = (A + B) \text{ and } u = h(A - B)$$

- The equation (1.22) gives the solution of the differential equation. In equation (1.22) as t increases the factor e^{-bt} decreases and [S + ut] increases. Therefore the displacement approaches zero as t increases. Such a motion is called **Critically Damped**.
- The example is pointer of voltmeter and current meter which comes to rest without oscillation.

Case III : Under Damped :

When $b^2 < \omega^2$, the component $\sqrt{(b^2 - \omega^2)}$ is imaginary.

The solution will be given by,

$$y = Ae^{(-b + i\beta)t} + Be^{(-b - i\beta)t}$$

$$\text{where,} \quad \beta = \sqrt{(\omega^2 - b^2)}$$

$$\text{and} \quad i = \sqrt{(-1)}$$

$$y = e^{-bt}(Ae^{i\beta t} + Be^{-i\beta t})$$

$$y = e^{-bt}[A(\cos\beta t + i\sin\beta t) + B(\cos\beta t - i\sin\beta t)]$$

$$y = e^{-bt}[(A + B)\cos\beta t + i(A - B)\sin\beta t]$$

$$\text{Let} \quad A + B = a\sin\phi$$

$$\text{and} \quad i(A - B) = a\cos\phi$$

$$\therefore \quad y = e^{-bt}(a\sin\phi\cos\beta t + a\cos\phi\sin\beta t)$$

$$y = e^{-bt}a\sin(\beta t + \phi) \quad \ldots (1.23)$$

- The equation represents the simple harmonic motion with amplitude ae^{-bt}. The amplitude of motion will continuously decrease because of the factor e^{-bt}. The factor e^{-bt} is called the damping factor and b the damping coefficient.
- The decay in the amplitude is decided by the damping factor and the oscillation is called **Under Damped**.
- Example of under damped oscillation is pendulum in air, electric oscillator etc. Fig. 1.2 shows under damped oscillation.

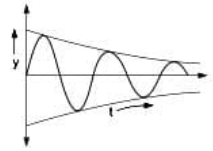


Fig. 1.2 : Under damped oscillation

1.4 FORCED OSCILLATION AND RESONANCE

1.4.1 Forced Oscillation

[May 18]

- Till now we have discussed free vibrations in which the body vibrates at its own frequency without any external force. But the situation will be totally different when the body is subjected to an external force. Here the body oscillates because it is subjected to an external periodic force. Such oscillation is called **Forced Oscillation**.

- A forced oscillation can be defined as the oscillation in which a body vibrates with a frequency other than its natural frequency under the action of an external periodic force.

In a forced oscillation, the forces acting on the body are.

(i) Restoring force, – kx.

(ii) Resistive force, $-\frac{rdy}{dt}$

(iii) External periodic force, f sin pt

The sum of above forces is balanced by Newton's force,

i.e. $$ma = -ky - \frac{rdy}{dt} + f\sin pt \qquad \ldots (1.24)$$

$$\therefore \quad m\frac{d^2y}{dt^2} = -ky - \frac{rdy}{dt} + f\sin pt$$

$$m\frac{d^2y}{dt^2} + \frac{rdy}{dt} + ky = f\sin pt$$

$$\frac{d^2y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{f}{m}\sin pt$$

Taking $\frac{r}{m} = 2b, \frac{k}{m} = \omega^2$ and $\frac{f}{m} = f$

$$\therefore \frac{d^2y}{dt^2} + 2b\frac{dy}{dt} + \omega^2 y = f\sin pt \qquad \ldots (1.25)$$

The equation is differential equation of motion of particle.

- At a steady state the body oscillates with the frequency of applied force and not with its natural frequency. The solution of equation (1.25) will be of the form,

$$y = A\sin(pt - \theta) \qquad \ldots (1.26)$$

where, A and θ are arbitrary constants, Differentiating equation (1.26),

$$\frac{dy}{dt} = Ap\cos(pt - \theta)$$

and $$\frac{d^2y}{dt^2} = -Ap^2\sin(pt - \theta)$$

Substituting in equation (1.25)

$$-Ap^2\sin(pt - \theta) + 2b\,Ap\cos(pt - \theta) + \omega^2 A\sin(pt - \theta)$$

$$= f\sin pt = f\sin[(pt - \theta) + \theta]$$

$$A(\omega^2 - p^2)\sin(pt - \theta) + 2b\,Ap\cos(pt - \theta)$$

$$= f\sin(pt - \theta)\cos\theta + f\cos(pt - \theta)\sin\theta$$

comparing coefficients of sin (pt – θ) and cos (pt – θ) on both sides, we get,

$$A(\omega^2 - p^2) = f\cos\theta \qquad \ldots (1.27)$$

$$2b\,Ap = f\sin\theta \qquad \ldots (1.28)$$

Squaring and adding equations (1.27) and (1.28),

$$A^2(\omega^2 - p^2)^2 + 4b^2A^2p^2 = f^2(\cos^2\theta + \sin^2\theta)$$

$$A^2[(\omega^2 - p^2)^2 + 4b^2p^2] = f^2$$

$$\therefore \quad A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}} \qquad \ldots (1.29)$$

Dividing equation (1.28) by (1.27), we get,

$$\tan\theta = \frac{2b\,Ap}{A(\omega^2 - p^2)}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) \qquad \ldots (1.30)$$

Equation (1.29) gives the amplitude and equation (1.30) phase of oscillations.

Depending upon the relative values of p and ω we have following cases.

Case I : When driving frequency is low i.e. p << ω.

In this case, the amplitude of oscillation is given by,

$$A = \frac{f}{\sqrt{[(\omega^2 - p^2)^2 + 4b^2p^2]}}$$

$$= \frac{f}{\sqrt{\omega^4}} = \frac{f}{\omega^2} = \text{constant}$$

and $$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}(0) = 0$$

The amplitude depends on the magnitude of applied force and force and displacement are in phase.

Case II : When p = ω i.e. frequency of the force is equal to the frequency of the body.

In this case the amplitude of oscillation is given by,

$$A = \frac{f}{2bp} = \frac{f}{r\omega}$$

and $$\theta = \tan^{-1}\left(\frac{bp}{0}\right) = \tan^{-1}(\infty) = \pi/2$$

Thus the amplitude of oscillation will depend on the damping force and the amplitude will be very large. The displacement and the force will have a phase difference of π/2.

Case III : When $p >> \omega$ i.e. the frequency of the force is greater than natural frequency ω of the body.

The amplitude in this case,

$$A = \frac{f}{\sqrt{p^2 + 4b^2 p^2}} = \frac{f}{p^2} = \frac{f}{mp^2}$$

and
$$\theta = \tan^{-1}\left(\frac{2bp}{\omega^2 - p^2}\right) = \tan^{-1}\left(\frac{2bp}{-p^2}\right)$$
$$= \tan^{-1}(0) = \pi$$

Thus the amplitude A decreases and the phase difference tends to π.

1.4.2 Resonance

- In case of forced oscillation, a body vibrates with the frequency of the external force causing the oscillation rather than its natural frequency. The resultant amplitude under forced vibration is given by equation.

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}} \quad \text{... (1.31)}$$

- From above equation it is clear that the resultant amplitude of oscillation varies with the frequency value of force p. For a particular value of p the amplitude becomes maximum. This phenomenon is known as **Resonance**.
- Thus, phenomenon of making a body oscillate with its natural frequency under the influence of another oscillating body with the same frequency is called resonance. For amplitude to be maximum, $\sqrt{[(\omega^2 - p^2)^2 + 4b^2 p^2]}$ has to be minimum.

i.e. $\frac{d}{dp}[(\omega^2 - p^2)^2 + 4b^2p^2] = 0$

$$2(\omega^2 - p^2)(-2p) + 4b^2(2p) = 0$$

$$\therefore \quad \omega^2 - p^2 = 2b^2$$

or
$$p = \sqrt{(\omega^2 - 2b^2)} \quad \text{... (1.32)}$$

If the damping is small i.e. b is negligible, then above equation reduces to

$$p = \omega \quad \text{... (1.33)}$$

which is the condition for resonance.

Substituting this condition in the equation (1.31) we get,

$$A_{max} = \frac{f}{\sqrt{(p^2 - p^2)^2 + 4b^2p^2}}$$
$$= \frac{f}{2bp} \quad \text{... (1.34)}$$

Thus A_{max} approaches to infinity when damping force approaches to zero.

Sharpness of Resonance :

- The amplitude of forced oscillation is maximum when the frequency of the applied force satisfies the condition of resonance i.e. $p = \sqrt{(\omega^2 - 2b^2)}$. If the frequency changes from this value the amplitude falls. The rate of fall in the amplitude with the change of forcing frequency on each side of the resonance frequency is called sharpness of the resonance.
- Fig. 1.3 shows the variation of amplitude with the forcing frequency at different damping values.

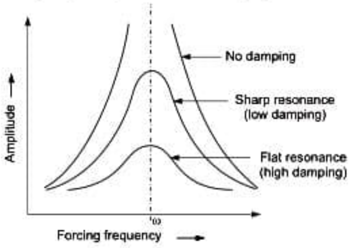


Fig. 1.3 : Resonance

1.5 DIFFERENTIAL WAVE EQUATION [Dec. 17]

- A wave motion is a disturbance which travels through available space or medium and the medium particle vibrates around their mean position when the wave approaches. The motion is handed over from one particle to the next after regular interval of time.
- Consider a progressive wave originating at the origin O and travelling along the positive x - axis with velocity v as shown in Fig. 1.4.

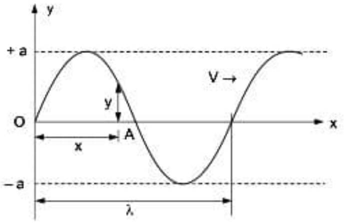
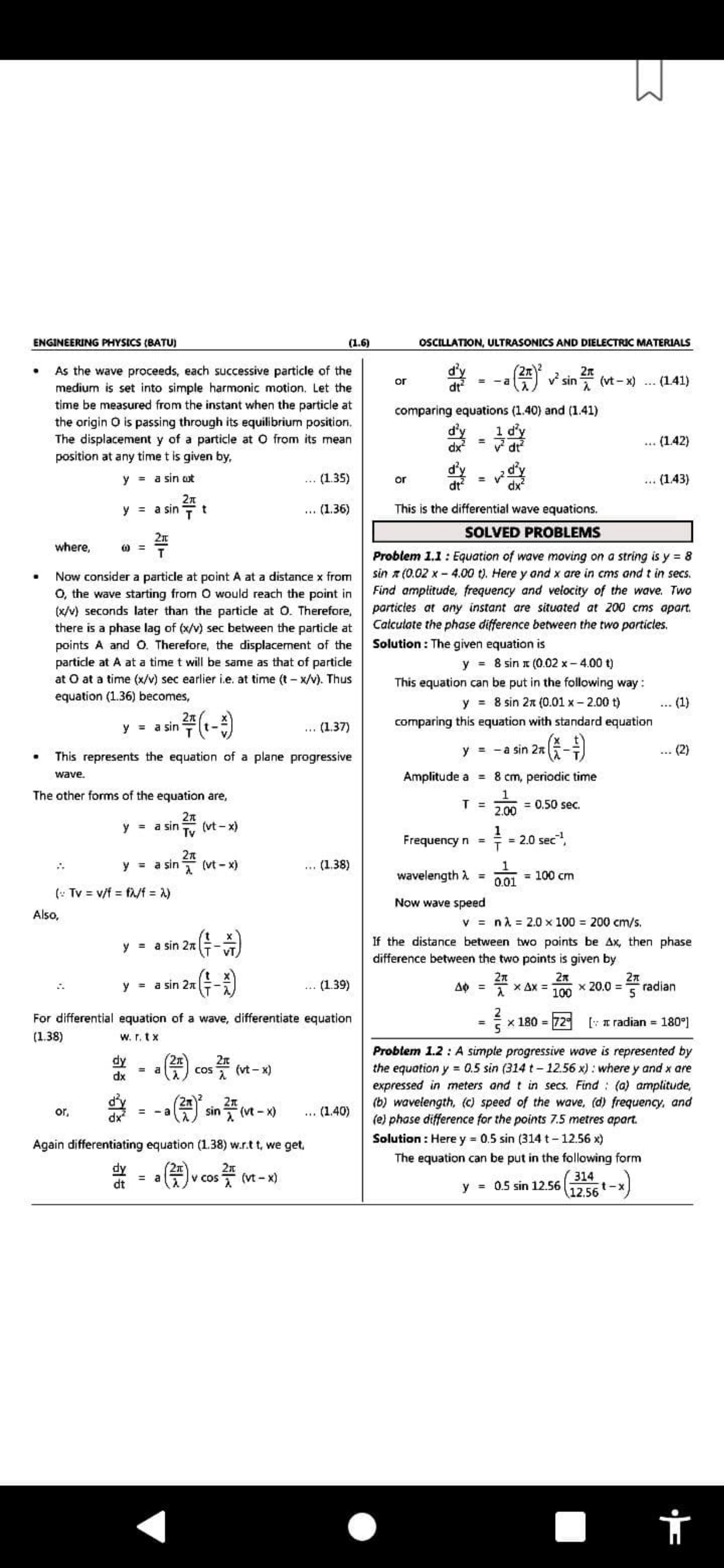


Fig. 1.4 : Progressive wave



- As the wave proceeds, each successive particle of the medium is set into simple harmonic motion. Let the time be measured from the instant when the particle at the origin O is passing through its equilibrium position. The displacement y of a particle at O from its mean position at any time t is given by,

$$y = a \sin \omega t \qquad \dots (1.35)$$

$$y = a \sin \frac{2\pi}{T} t \qquad \dots (1.36)$$

where, $\omega = \frac{2\pi}{T}$

- Now consider a particle at point A at a distance x from O, the wave starting from O would reach the point in (x/v) seconds later than the particle at O. Therefore, there is a phase lag of (x/v) sec between the particle at points A and O. Therefore, the displacement of the particle at A at a time t will be same as that of particle at O at a time (x/v) sec earlier i.e. at time (t – x/v). Thus equation (1.36) becomes,

$$y = a \sin \frac{2\pi}{T}\left(t - \frac{x}{v}\right) \qquad \dots (1.37)$$

- This represents the equation of a plane progressive wave.

The other forms of the equation are,

$$y = a \sin \frac{2\pi}{Tv} (vt - x)$$

$$\therefore \quad y = a \sin \frac{2\pi}{\lambda} (vt - x) \qquad \dots (1.38)$$

($\because$ Tv = v/f = fλ/f = λ)

Also,

$$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{vT}\right)$$

$$\therefore \quad y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \qquad \dots (1.39)$$

For differential equation of a wave, differentiate equation (1.38) w. r. t x

$$\frac{dy}{dx} = a\left(\frac{2\pi}{\lambda}\right) \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\text{or,} \quad \frac{d^2y}{dx^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 \sin \frac{2\pi}{\lambda} (vt - x) \qquad \dots (1.40)$$

Again differentiating equation (1.38) w.r.t t, we get,

$$\frac{dy}{dt} = a\left(\frac{2\pi}{\lambda}\right) v \cos \frac{2\pi}{\lambda} (vt - x)$$

$$\text{or} \quad \frac{d^2y}{dt^2} = -a\left(\frac{2\pi}{\lambda}\right)^2 v^2 \sin \frac{2\pi}{\lambda} (vt - x) \qquad \dots (1.41)$$

comparing equations (1.40) and (1.41)

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \qquad \dots (1.42)$$

$$\text{or} \quad \frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \qquad \dots (1.43)$$

This is the differential wave equations.

SOLVED PROBLEMS

Problem 1.1 : *Equation of wave moving on a string is y = 8 sin π (0.02 x – 4.00 t). Here y and x are in cms and t in secs. Find amplitude, frequency and velocity of the wave. Two particles at any instant are situated at 200 cms apart. Calculate the phase difference between the two particles.*

Solution : The given equation is

$$y = 8 \sin \pi (0.02\,x - 4.00\,t)$$

This equation can be put in the following way :

$$y = 8 \sin 2\pi (0.01\,x - 2.00\,t) \qquad \dots (1)$$

comparing this equation with standard equation

$$y = -a \sin 2\pi \left(\frac{x}{\lambda} - \frac{t}{T}\right) \qquad \dots (2)$$

Amplitude a = 8 cm, periodic time

$$T = \frac{1}{2.00} = 0.50 \text{ sec.}$$

$$\text{Frequency } n = \frac{1}{T} = 2.0 \text{ sec}^{-1},$$

$$\text{wavelength } \lambda = \frac{1}{0.01} = 100 \text{ cm}$$

Now wave speed

$$v = n\,\lambda = 2.0 \times 100 = 200 \text{ cm/s.}$$

If the distance between two points be Δx, then phase difference between the two points is given by

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{100} \times 20.0 = \frac{2\pi}{5} \text{ radian}$$

$$= \frac{2}{5} \times 180 = \boxed{72°} \qquad [\because \pi \text{ radian} = 180°]$$

Problem 1.2 : *A simple progressive wave is represented by the equation y = 0.5 sin (314 t – 12.56 x) : where y and x are expressed in meters and t in secs. Find : (a) amplitude, (b) wavelength, (c) speed of the wave, (d) frequency, and (e) phase difference for the points 7.5 metres apart.*

Solution : Here y = 0.5 sin (314 t – 12.56 x)

The equation can be put in the following form

$$y = 0.5 \sin 12.56 \left(\frac{314}{12.56} t - x\right)$$

Comparing this equation with the standard equation

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

(a) amplitude $a = 0.5$ meters

(b) $\frac{2\pi}{\lambda} = 12.56$ i.e., $\lambda = \frac{2\pi}{12.56} = 0.5$ meters

(c) velocity $v = \frac{314}{12.56} = 25$ meters/sec.

(d) frequency $n = \frac{v}{\lambda} = \frac{25}{0.5} = \boxed{\text{50 per sec}}$

Problem 1.3 : *A train of simple harmonic wave traveling in a gas along the positive direction of X axis with an amplitude 2 cm velocity 45 metres per sec. and frequency 75. Calculate the displacement, particle velocity and acceleration at a distance of 135 cm from the origin after an interval of 3 sec.*

Solution : The displacement of a particle y in a plane progressive wave is given by

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

Here $a = 2$ cm, $v = 4500$ cm/sec, $n = 75$,

$$x = 135 \text{ cm}, t = 3 \text{ sec}$$

and

$$\lambda = \frac{v}{n} \times \frac{4500}{75} = 60 \text{ cm}$$

$$\therefore \quad y = 2 \sin \frac{2\pi}{60} (4500 \times 3 - 135)$$

$$= 2 \sin \frac{891\pi}{2} = 2 \sin \left(445\pi + \frac{\pi}{2}\right)$$

$$= -2 \sin \pi/2 = -2 \text{ cm}$$

The particle velocity

$$\frac{dy}{dt} = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda} (vt - x)$$

As above $\sin \frac{2\pi}{\lambda} (vt - x) = -1$

$$\therefore \cos \frac{2\pi}{\lambda} (vt - x) = 0$$

Hence, $\frac{dy}{dt} = 0.0$ cm/sec.

Particle acceleration,

$$\frac{d^2y}{dt^2} = -\frac{4\pi^2 v^2}{\lambda^2} a \sin \frac{2\pi}{\lambda} (vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2} y = -\frac{4\pi^2 (4500)^2}{(60)^2} \times (-2)$$

$$= \boxed{4.437 \times 10^5 \text{ cm/sec}^2}$$

1.6 ULTRASONIC WAVES [May 18]

- We all know that sound is due to the **Vibrations of Particles of the Medium**. Human ear can hear the sound waves of frequencies between 20 Hz to 20,000 Hz. This range of frequencies is known as the **Audible Range**.
- The sound waves whose frequencies are greater than 20,000 Hz are known as **Ultrasonic Waves**. The wavelength of ultrasonic waves is very small as compared to that of audible sound. The sound waves which have frequencies less than the audible range are called as **Infrasonic Waves**.
- The ultrasonic and infrasonic frequencies are inaudible to human beings but they are audible to some birds, dogs and insects. A dog can hear sound of frequencies above 20 kHz. Bats can hear sound waves of frequencies upto 100 kHz. This enables them to move freely even in the dark.
- As the ultrasonic waves have very high frequency they undergo very less diffraction and therefore less spreading. Because of this property, high energy can be concentrated in a very narrow beam and can cover very large distances with very less losses of energy. This special feature makes them useful in many applications.

1.7 PRODUCTION OF ULTRASONIC WAVES

- Ultrasonic waves cannot be produced by the ordinary method i.e. by using mechanical vibrations. This is because of the comparatively low natural frequencies of the moving parts. Hence other methods are used for the production of ultrasonic waves. The method chosen depends upon the output power required and the frequency range needed.
- A device which produces ultrasonic waves is called an **Ultrasonic Transducer**. To generate lower frequencies, a mechanical type device such as Galton's Whistle is used. Magnetostriction method is used when frequencies upto 300 kHz are needed, while piezo-electric generators are used mostly for frequencies above that.

1.7.1 Piezo-Electric Effect [Dec. 18, May 19]

- When opposite faces of a thin section of certain crystals like tourmaline, quartz etc. are subjected to distortion by applying **Pressure or Tension**, then **Equal and Opposite Charges** are developed on the faces perpendicular to the faces subjected to distortion. The magnitude of the potential difference developed is proportional to the amount of distortion produced.

The polarity of the charges gets reversed when the direction of the force of distortion is reversed. This phenomenon is known as **Piezo-Electric Effect**.

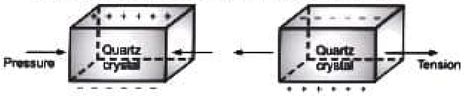


Fig. 1.5: Piezo-electric effect

- The converse of piezo-electric effect is also true i.e. if a potential difference is applied across the two faces of the crystal, it expands or contracts depending on the strength and direction of the applied field.
- Instead of steady voltage if an alternating voltage is applied across the faces of the crystal, then the crystal will expand and contract alternatively. This alternate expansion and contraction will make the crystal vibrate.
- If the frequency of the applied a.c. voltage happens to be equal to one of the modes of vibration of the crystal, resonance occurs and the crystal vibrates with maximum amplitude.

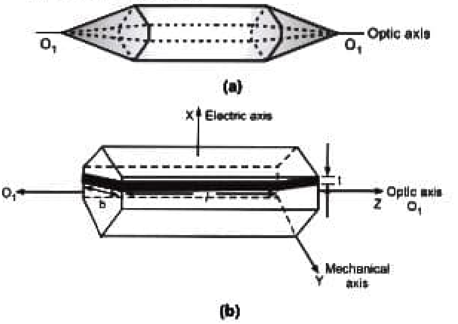


Fig. 1.6: (a) Natural quartz crystal and (b) Transverse section of quartz crystal cut along a plane perpendicular to the optic axis

Fig. 1.6 (a) shows a natural quartz crystal and Fig. 1.6 (b) shows a transverse section of the crystal.

- Consider a quartz crystal plate of thickness t and length l (along the optic axis). When an alternating voltage is applied across the faces of this crystal along the electrical axis, then alternating stress and strain is set up both in its thickness and length.
- If the frequency of the alternating voltage coincides with the natural frequency of vibration of the crystal, resonance occurs. The crystal vibrates with large amplitude. On maintaining suitable alternating potential, ultrasonic waves can be generated by this method.
- The frequency of the thickness vibrations,

$$f = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

- The frequency of the length vibrations,

$$f = \frac{P}{2l}\sqrt{\frac{E}{\rho}}$$

where P = 1, 2, 3, ... etc. for **Fundamental**, **First Overtone** and **Second Overtone** respectively, E is the Young's modulus, ρ is the density of the crystal, t is the thickness and l is the length of the crystal.

Piezo-Electric Oscillator [Dec. 18]

- The experimental set up is as shown in Fig. 1.7.
- The high frequency alternating voltage applied to the crystal is obtained from an oscillatory circuit (inductance L_1 and a variable condenser C_1 in parallel).
- One end of the oscillatory circuit is connected to the plate of the valve and the other end to the grid. The quartz crystal is placed in between two metal plates A and B to form a parallel-plate capacitor with the crystal as a dielectric. This is connected in parallel to the variable condenser C_1.
- By adjusting the variable condenser, the frequency of the oscillatory circuit is tuned to the natural frequency of the crystal.
- At this stage, the crystal is set into mechanical vibrations and ultrasonic waves are generated. By this method, ultrasonic waves upto a frequency of 15×10^7 Hz can be obtained.

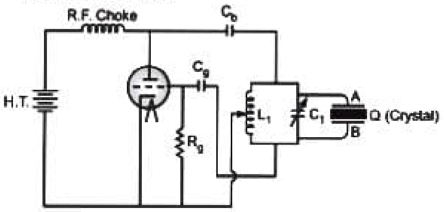


Fig. 1.7: Piezo-electric oscillator

Alternative Method for Piezo-electric Oscillator

Construction

- The piezo-electric oscillator uses basically a Hartley oscillator. The transistor is biased using the resistors R_1, R_2 and R_E. The combination of L_1, L_2 and C_1 works as tuning circuit which is couple with the transistor with a coupling capacitor C_2. The capacitor C_2 is used to provide positive feedback.

- The resonance frequency of the tank circuit is given by

$$f_r = \frac{1}{2\pi\sqrt{L_r C_1}}, \qquad \text{where } L_r = L_1 + L_2$$

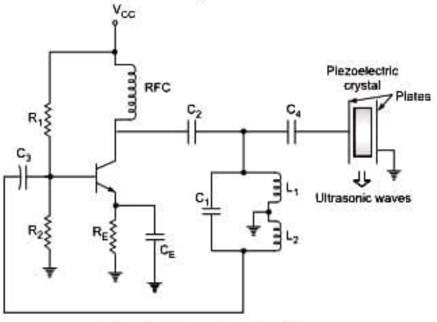


Fig. 1.8 : Piezo-electric oscillator

Working

- When the circuit is switched on, oscillating currents are produced in the tuning circuit.
- The oscillating currents generated by the tuning circuit are sustained and the electric signal obtained at the output is applied to the piezoelectric crystal through coupling capacitor C_4.
- When these high frequency electrical signals are applied to the crystal, because of reverse piezoelectric effect, the crystal undergoes alternate contraction and expansion. These vibrations produce ultrasonic waves.
- The frequency of ultrasonic waves can be changed by varying the values of components of the tuning circuit as per the relation $f = \frac{1}{2\pi\sqrt{L_T C_1}}$. When frequency of oscillation of the tuning circuit becomes equal to the natural frequency of the crystal $f = \frac{P}{2l}\sqrt{\frac{E}{r}}$, resonance occurs and crystal oscillates with maximum amplitude and amplitude of ultrasonic waves will be maximum.
- Ultrasonic waves upto frequency of 1.5×10^8 Hz can be produced using piezoelectric oscillator.

1.7.2 Magnetostriction Effect [Dec. 17, May 18]

- According to this phenomenon, a rod of **Ferromagnetic Material** such as iron or nickel undergoes a **Change in its Length** when placed in a magnetic field parallel to its length.
- Instead of a steady field, if an alternating field is used, the rod expands and contracts in length alternately. This sets up a longitudinal vibration in the rod whose frequency is twice the frequency of the alternating magnetic field. If the natural frequency of the rod and the frequency of the alternating field is the same, resonance occurs and the amplitude of vibration of the rod is maximum.
- The range of frequency depends on the dimensions of the magneto strictive material. The longitudinal vibrations thus produced are exactly like those produced by a rod which is clamped at the mid point but has both ends free.
- The frequency of vibration of such a rod is

$$\boxed{f = \frac{1}{2l}\sqrt{\frac{E}{\rho}}}$$

where E is the Young's modulus,

l is the length of the rod, and

ρ is its density of rod.

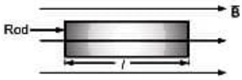


Fig. 1.9: Magnetostriction effect

Magnetostriction Oscillator

- This apparatus generates ultrasonic waves and is based on the principle of magnetostriction.
- Fig. 1.10 shows the experimental set up of magnetostriction oscillator. It consists of a permanently magnetised nickel or iron rod (magnetised initially by passing direct current in the coil which is wound around it). The rod is clamped at the centre. The two coils L_1 and L_2 are wound over the rod.
- The exciting coil L_1 is connected to the plate circuit of a valve while the coil L_2 is coupled to the plate via the grid circuit.
- By adjusting the variable condenser C, high frequency oscillation currents are set up in the plate circuit. This high frequency current flowing through the coil L_1 produces changes in the length of the rod. Due to this, the rod expands and contracts alternately and a vibration is set up in the rod.

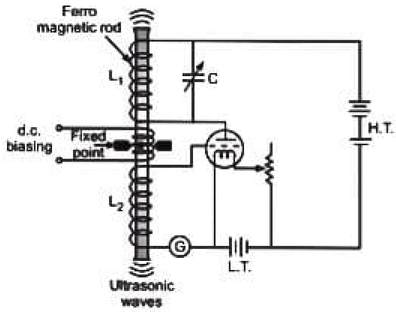


Fig. 1.10: Magnetostriction oscillator

- These vibrations in the length of the rod cause a variation in the magnetic flux through the coil L_2 and an e.m.f. is induced in it. This induced e.m.f. is fed to the grid which produces large variations in the plate current. Thus magneto strictive effect in the bar is increased.
- When the frequency of the circuit becomes equal to the natural frequency of the rod, resonance occurs and ultrasonic waves of maximum amplitude are produced. By adjusting the length of the rod and condenser capacity, high frequency oscillations of different frequencies can be obtained.

Alternate Method for Magnetostriction Oscillator

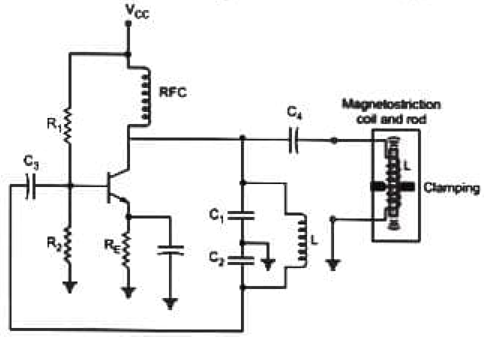


Fig. 1.11

Construction

- The magneto strictive oscillator uses basically a Colpitt's oscillator. The transistor is biased with the help of resistances R_1, R_2 and R_E. The inductance L and capacitors C_1 and C_2 form a tank circuit and C_3 is a feedback capacitor. The tank circuit is used for selecting resonance frequency.
- The appropriate frequencies at the end of the tank circuit are amplified and the oscillations corresponding to them are sustained. The resonance of tank circuit is given by $f_r = \frac{1}{2\pi\sqrt{LC_r}}$, where $C_r = \frac{C_1C_2}{C_1 + C_2}$. A ferromagnetic rod is kept surrounding to a coil in the shape of solenoid at the output of the oscillator.

Working

- When the circuit is switched on, oscillating currents are produced in the tuning circuit. The oscillations appearing at output terminal of oscillator circuit are fed to the magnetostriction coil through the coupling capacitor C_4. The magnetostriction coil is placed surrounding the ferromagnetic rod.
- The coil produces magnetic field which is alternately changing in opposite directions and is applied around the ferromagnetic rod. Due to magnetostriction effect, the changing magnetic field causes rod to contract and expand alternately. These vibrations of the rod travel in surrounding medium in the form of ultrasonic waves.
- The frequency of oscillating current in the tank circuit can be changed by varying the values of the components of the tank circuit.
- The frequency of vibrations of the rod is given by $f = \frac{p}{2l}\sqrt{\frac{E}{\rho}}$.
- where, p is integer, l is length of the rod, Y is Young's modulus and ρ is density of the rod.
- When frequency of the tuning circuit becomes equal to natural frequency of the rod, the rod vibrates with maximum amplitude and ultrasonic waves with maximum amplitude are obtained.

1.8 PROPERTIES OF ULTRASONIC WAVES

- As the wavelength of the waves is very small, ultrasonic waves suffer least diffraction. They can be **Transmitted Over Longer Distances** as a highly directional beam without appreciable loss of energy.
- Ultrasonic waves are **Highly Energetic** and may have intensities upto 10 kW/m².
- On passing through liquids, ultrasonic waves are propagated longitudinally forming nodes and antinodes. This produces **Cavitation Effect.**
- In solids, ultrasonic waves propagate both longitudinally and transversely. In transverse waves, there exist no nodes or antinodes.

- Velocity of ultrasonic waves depends on the **Temperature of the Medium** through which it is propagating.
- When ultrasonic waves are passed through a liquid kept in a rectangular vessel, they are reflected from the bottom of the vessel. The directed and reflected rays get superimposed resulting in a stationary wave. Due to the formation of the stationary wave the density of the node is greater than that at the antinode. Now, if a parallel beam of light is passed at right angle to the wave the liquid acts as a diffraction grating. This is called as **Acoustical Grating.**

1.9 APPLICATIONS OF ULTRASONIC WAVES

1.9.1 Scientific Applications

- **Echo Sounding:** Ultrasonic sound waves are used for sound signalling, depth sounding, determining the position of ice bergs, submarines etc. These applications make use of the **echo principle**. The high frequency sound waves can be readily formed into a narrow beam and can be focused in any desired direction. Because of this, these waves can travel many kilometres in water before being absorbed. Ultrasonic waves of 50 kHz frequency are generated by a crystal vibrator. The moment signal is sent from the transmitter, a deflection of the spot on the C.R.O. screen is observed. The beam travels to the receiver or obstacle and reflects back. When the reflected beam returns, it is indicated by a deflection of the spot on the CRO. The time interval between the two deflections can be measured. Knowing the velocity of the ultrasonic waves and the time interval, the position of the receiver or obstacle can be determined. This is the underlying principle in echo sounding.
- **Depth Sounding:** This application of ultrasonic waves makes use of the **Echo Principle**. The depth of sea or the depth of water below a ship can be calculated using ultrasonic waves. Because of their high frequency and short wavelengths, ultrasonic waves are not absorbed by water so strongly as lower frequency waves. Waves of frequency of about 40 kHz are used. They are produced by a crystal transducer and are directed towards the bottom of the sea, at regular intervals. The reflected waves from the bottom of the sea are received by the same crystal causing it to vibrate. The vibrations generate a small e.m.f. across its faces which is recorded on a sensitive CRO. The time interval between the emission of initial wave pulse and the e.m.f. generated due to the reflected waves is recorded. If v is the velocity of ultrasonic waves in sea water and t is the time interval between sending and receiving of the wave, then

$$\text{Depth of sea} = \frac{\text{Velocity of sound in sea} \times \text{time}}{2}$$

$$d = \frac{vt}{2}$$

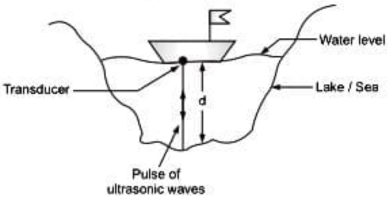


Fig. 1.12

- **Determining the Position of Icebergs, Submarines, a Shoal of Fish in Sea, etc.:** The principle employed here is the **echo principle**. Ultrasonic waves are reflected from objects even if they are very small. Hence the presence of submerged objects in the sea like icebergs, submarines etc. can be detected. Pulses of ultrasonic signals are sent out at short intervals. The reflected echo is received in the ultrasonic receiver and the time interval between the transmitted and received signals is noted. This is the two way travel time from the source to the target. Knowing the velocity of sound in water, the distance between the source and the target can be calculated.
- **Cleaning and Removing Dirt:** Clothes or utensils that have to be cleaned are subjected to ultrasonic waves. These waves will put the dirt particles or water particles into vibration. As a result, these particles loosen their attachment with the surface and fall-off. The same treatment is used for removing soot and dust from the chimney.

1.9.2 Engineering Applications

- **Non-Destructive Testing**
  - Non-destructive testing is characterized by low intensity of the sound wave used. Here sound wave is not expected to cause any change in the chemical or physical characteristics of the specimen material.
  - Such applications are found in testing, inspection and quality control.

- ➢ In this case, ultrasonic waves are propagated into the specimen under inspection. When the ultrasonic waves are incident on the defect, reflection of the wave from the interface (between material and defect) in the object takes place. Thus, the defects are located without any real damage to the specimen.
- ➢ Ultrasonic waves may be used for a large number of non-destructive testing on different materials. Some of these are
  - (i) Ultrasonic flaw detection
  - (ii) Ultrasonic study of structure of matter.

- **Flaw Detection**
  - ➢ The strength of components plays a significant role in most of the engineering applications. Any kind of defect greatly reduces the strength of materials. These defects can be as large as cracks or as tiny as cavities.
  - ➢ A high frequency pulse from pulse generator is impressed on a quartz crystal which is placed on the specimen under test. The crystal (transducer) first acts as a transmitter sending out high frequency waves into the specimen.
  - ➢ Then it acts as a receiver to receive the ultrasonic echo pulses reflected from the flaw and from the far end of the specimen. The received ultrasonic echo pulses are transformed by the transducer into corresponding electric echo pulses of the same frequency. These are then amplified and displayed on the C.R.O. screen as a series of pulses.
  - ➢ The first pulse corresponds to the transmitted wave, the next pulse corresponds to the reflected wave. i.e. first one from the flaw and the second one from the far end of the specimen. Each reflected pulse is indicated at a particular time after the initial transmitted pulse. The time interval between the transmitted and reflected pulse represents the distance travelled by the wave. From this the exact position of the flaw is located.

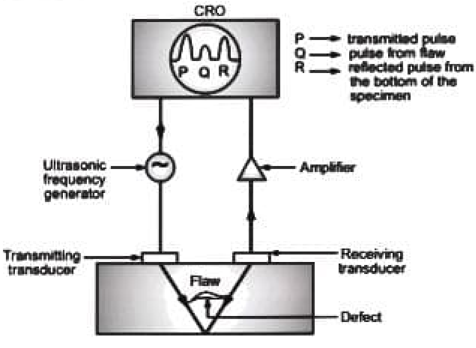


Fig. 1.13

- **Cavitation**
  - ➢ When an ultrasonic transducer is placed in a liquid, it produces ultrasonic vibrations in it. This results in the development and implosion of bubbles. These bubbles are known as the ***cavitation bubbles*** and they are formed as follows.
  - ➢ When a liquid is subjected to a powerful ultrasonic radiation, tension develops at some point in a liquid. The excess stress tears apart the liquid producing a hollow bubble that sucks in dissolved gases and vapours.
  - ➢ The life time of the bubble is very short, and it collapses very quickly. During the implosion of the bubble, the pressure of the shock wave that is formed near the bubble reaches several hundreds of atmospheres.
  - ➢ The formation and implosion of bubbles accounts for erosion and pitting of an ultrasonic transducer kept in the liquid. The bubbles have two effects : (i) they produce a dense cloud in front of the transducer and block the propagation of ultrasonic waves, (ii) frequent implosion of bubbles destroys the surface of the transducer causing pits.
  - ➢ Even though cavitation bubbles formed by ultrasonic vibrators in liquids block the wave propagation, it has some successful industrial applications like ultrasonic cleaning, ultrasonic emulsification etc.

- **Ultrasonic Cleaning**
  - ➢ Ultrasonic cleaning is achieved through a combined effect of cavitation and acceleration of

the cleaning liquid. Ultrasonic waves in liquid produce cavitation i.e. tiny space in the liquid. The vacuum created in these spaces exerts a strong pull on exposed solid surfaces. This detaches any particles of dust attached to them.

- The transducer is placed at the bottom of the tank in which the cleaning solution (either a water detergent solution or standard solvents) is taken.
- For the cleaning of metallic parts, low frequency waves are used while for cleaning fibres, high frequency waves are used. The specimen to be cleaned is kept immersed in the cleaning solution in the tank.

- **Ultrasonic Emulsification**
  - It has been observed that intense ultrasonic waves can thoroughly mix immiscible liquids like oil and water to form a stable emulsion. The emulsification results because of the cavitation bubbles imploding at the boundary surfaces between a liquid and vibrator, and also between liquid and walls of the container.
  - The two liquids which are to be emulsified are taken in a container. This container is placed in a liquid bath which is subjected to strong ultrasonic vibrations by a transducer. Then emulsification due to gas bubbles takes place at the surface of the container containing the two liquids.
- **Measurement Gauge**
  - Ultrasonic thickness measurement is based on the **Echo Principle**. A piezoelectric transducer attached to the test piece converts the electric pulse into ultrasonic waves. The transducer can be attached to the test piece directly or it can be coupled to the piece by an incompressible medium such as oil or water.
  - The ultrasonic waves propagate into the test piece. They travel through the sample and are reflected back from the opposite surface. The same transducer then receives the reflected echo and converts it to an electrical pulse. The time taken for the pulse to travel through the sample is related to the thickness and the velocity in the material. The thickness T can be expressed by the formula,

$$\text{Thickness} = \frac{\text{Velocity of sound in specimen} \times \text{time}}{2}$$

$$T = \frac{vt}{2}$$

where, v is the velocity of the ultrasound in the material and t is the time between the pulse being transmitted and the echo being received.

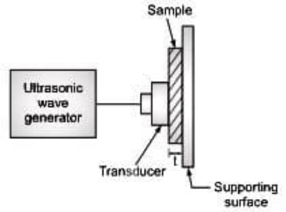


Fig. 1.14

- If the velocity of ultrasonic waves in the material is not known, then the value has to be determined experimentally by using the same material whose thickness is already known. From the known values of T and t, the velocity v of ultrasound in the material can be calculated.
- The advantage of using ultrasonic waves for thickness measurement is that the thickness can be measured from one side of the test piece. There is no need for drilling holes or otherwise inflicting damage to the piece.

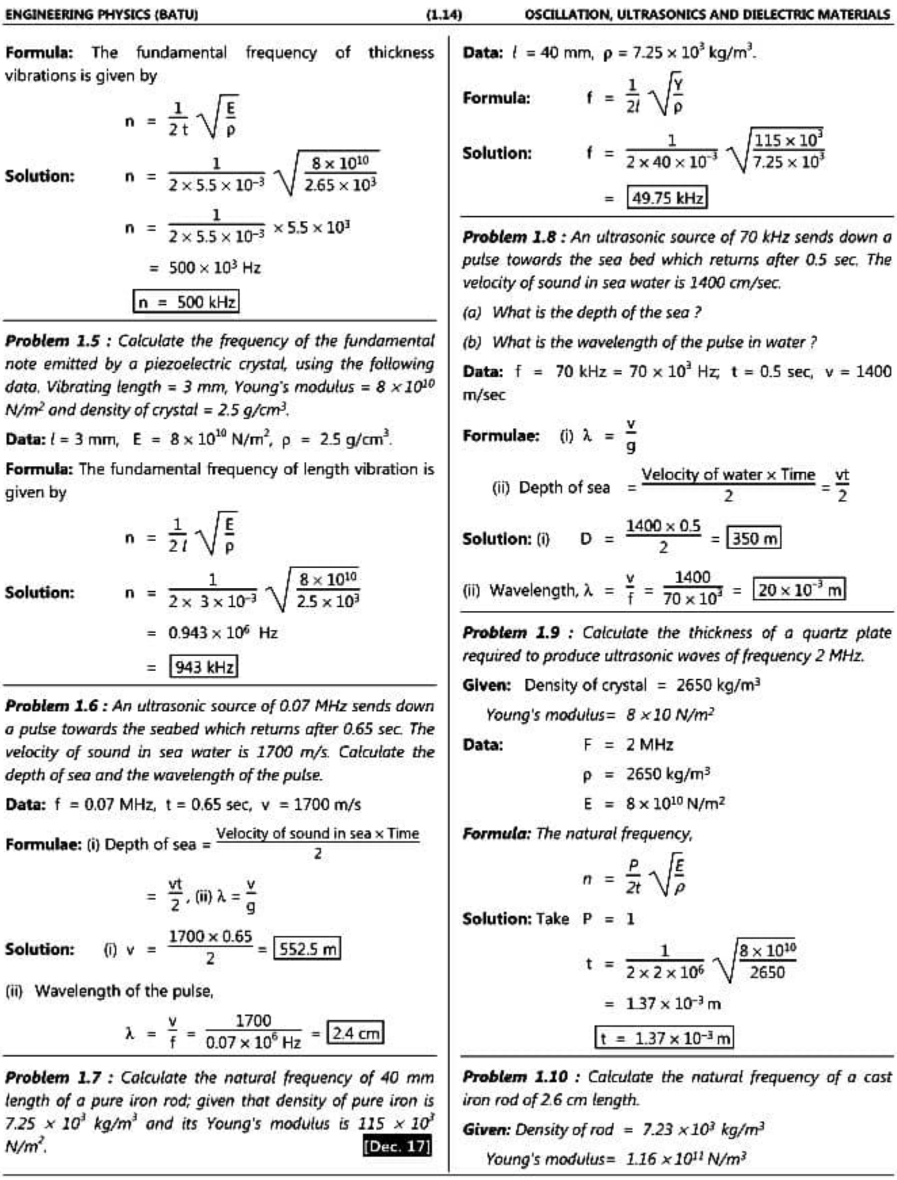
1.9.3 Medical Applications

Certain tumors which cannot be detected by X-rays can be detected by ultrasonic waves. Joints affected by rheumatic pains get great relief when exposed to ultrasonic waves. Surgical use of ultrasonic waves include the selective cutting of the tissue during an operation. Ultrasonic waves are very useful for dental cutting because

- They make the cutting almost painless.
- They cut the hard material very easily, and
- They do not require any mechanical device for cutting purpose. Thus, ultrasonography now has become an important tool which help physicians in diagnosing and treating medical ailments.

Problem 1.4 : *Calculate the natural frequency of the thickness vibrations for quartz plate of thickness 5.5×10^{-3} m, given that Young's modulus along X-axis is 8×10^{10} N/m^2 and density of crystal is 2.65×10^3 kg/m^3.*

Data: $t = 5.5 \times 10^{-3}$ m, $E = 8 \times 10^{10}$ N/m^2, $\rho = 2.65 \times 10^3$ kg/m^3.



Formula: The fundamental frequency of thickness vibrations is given by

$$n = \frac{1}{2t}\sqrt{\frac{E}{\rho}}$$

Solution:

$$n = \frac{1}{2 \times 5.5 \times 10^{-3}}\sqrt{\frac{8 \times 10^{10}}{2.65 \times 10^{3}}}$$

$$n = \frac{1}{2 \times 5.5 \times 10^{-3}} \times 5.5 \times 10^{3}$$

$$= 500 \times 10^{3} \text{ Hz}$$

$$\boxed{n = 500 \text{ kHz}}$$

Problem 1.5 : *Calculate the frequency of the fundamental note emitted by a piezoelectric crystal, using the following data. Vibrating length = 3 mm, Young's modulus = 8×10^{10} N/m^2 and density of crystal = 2.5 g/cm^3.*

Data: l = 3 mm, E = 8×10^{10} N/m^2, ρ = 2.5 g/cm^3.

Formula: The fundamental frequency of length vibration is given by

$$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution:

$$n = \frac{1}{2 \times 3 \times 10^{-3}}\sqrt{\frac{8 \times 10^{10}}{2.5 \times 10^{3}}}$$

$$= 0.943 \times 10^{6} \text{ Hz}$$

$$= \boxed{943 \text{ kHz}}$$

Problem 1.6 : *An ultrasonic source of 0.07 MHz sends down a pulse towards the seabed which returns after 0.65 sec. The velocity of sound in sea water is 1700 m/s. Calculate the depth of sea and the wavelength of the pulse.*

Data: f = 0.07 MHz, t = 0.65 sec, v = 1700 m/s

Formulae: (i) Depth of sea $= \frac{\text{Velocity of sound in sea} \times \text{Time}}{2}$

$$= \frac{vt}{2}, \text{ (ii) } \lambda = \frac{v}{g}$$

Solution: (i) $v = \frac{1700 \times 0.65}{2} = \boxed{552.5 \text{ m}}$

(ii) Wavelength of the pulse,

$$\lambda = \frac{v}{f} = \frac{1700}{0.07 \times 10^{6} \text{ Hz}} = \boxed{2.4 \text{ cm}}$$

Problem 1.7 : *Calculate the natural frequency of 40 mm length of a pure iron rod; given that density of pure iron is 7.25×10^{3} kg/m^3 and its Young's modulus is 115×10^{3} N/m^2.* **[Dec. 17]**

Data: l = 40 mm, ρ = 7.25×10^{3} kg/m^3.

Formula: $f = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$

Solution:

$$f = \frac{1}{2 \times 40 \times 10^{-3}}\sqrt{\frac{115 \times 10^{3}}{7.25 \times 10^{3}}}$$

$$= \boxed{49.75 \text{ kHz}}$$

Problem 1.8 : *An ultrasonic source of 70 kHz sends down a pulse towards the sea bed which returns after 0.5 sec. The velocity of sound in sea water is 1400 cm/sec.*

(a) What is the depth of the sea ?

(b) What is the wavelength of the pulse in water ?

Data: f = 70 kHz = 70×10^{3} Hz; t = 0.5 sec, v = 1400 m/sec

Formulae: (i) $\lambda = \frac{v}{g}$

(ii) Depth of sea $= \frac{\text{Velocity of water} \times \text{Time}}{2} = \frac{vt}{2}$

Solution: (i) $D = \frac{1400 \times 0.5}{2} = \boxed{350 \text{ m}}$

(ii) Wavelength, $\lambda = \frac{v}{f} = \frac{1400}{70 \times 10^{3}} = \boxed{20 \times 10^{-3} \text{ m}}$

Problem 1.9 : *Calculate the thickness of a quartz plate required to produce ultrasonic waves of frequency 2 MHz.*

Given: Density of crystal = 2650 kg/m^3

Young's modulus= 8×10 N/m^2

Data:

F = 2 MHz

ρ = 2650 kg/m^3

E = 8×10^{10} N/m^2

Formula: *The natural frequency,*

$$n = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

Solution: Take P = 1

$$t = \frac{1}{2 \times 2 \times 10^{6}}\sqrt{\frac{8 \times 10^{10}}{2650}}$$

$$= 1.37 \times 10^{-3} \text{ m}$$

$$\boxed{t = 1.37 \times 10^{-3} \text{ m}}$$

Problem 1.10 : *Calculate the natural frequency of a cast iron rod of 2.6 cm length.*

Given: *Density of rod = 7.23×10^{3} kg/m^3*

Young's modulus= 1.16×10^{11} N/m^3

Data: $l = 2.6$ cm $\rho = 7.23 \times 10^3$ kg/m^3 $E = 1.16 \times 10^{11}$ N/m^2

Formula:

$$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution:

$$n = \frac{1}{2 \times 2.6 \times 10^{-2}}\sqrt{\frac{1.16 \times 10^{11}}{7.23 \times 10^{3}}}$$

$$n = 770.29 \times 10^2 \text{ Hz} = \boxed{77.03 \text{ kHz}}$$

***Problem 1.11 :** A quartz crystal of thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency given that Y for quartz is 7.9 × 10^{10} N/m^2 and ρ for quartz is 2650 kg/m^3.*

Data: $t = 0.001$ m, $\lambda = 7.9 \times 10^{10}$ N/m^2, $\rho = 2650$ kg/m^3

Solution:

$$f = \frac{1}{2t}\sqrt{\frac{\lambda}{\rho}}$$

$$= \frac{1}{2 \times 0.001}\sqrt{\frac{7.9 \times 10^{10}}{2650}} = 2729.9 \text{ kHz}$$

$$= \boxed{2730 \text{ kHz}}$$

***Problem 1.12 :** Calculate the thickness of a quartz plate required to produce ultrasonic waves of frequency 2 MHz.*

Given : Density of crystal = 2650 kg/m^3

Young's modulus = 8×10 N/m^2

Data :

$F = 2$ MHz

$\rho = 2650$ kg/m^3

$E = 8 \times 10^{10}$ N/m^2

Formula : The natural frequency,

$$n = \frac{P}{2t}\sqrt{\frac{E}{\rho}}$$

Solution : Take $P = 1$

$$t = \frac{1}{2 \times 2 \times 10^6}\sqrt{\frac{8 \times 10^{10}}{2650}}$$

$$= 1.37 \times 10^{-3} \text{ m}$$

$$t = \boxed{1.37 \times 10^{-3} \text{ m}}$$

***Problem 1.13 :** Calculate the natural frequency of a cast iron rod of 2.6 cm length.*

Given :

Density of rod = 7.23×10^3 kg/m^3

Young's modulus = 1.16×10^{11} N/m^3

Data :

$l = 2.6$ cm

$\rho = 7.23 \times 10^3$ kg/m^3

$E = 1.16 \times 10^{11}$ N/m^2

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{E}{\rho}}$$

Solution :

$$n = \frac{1}{2 \times 2.6 \times 10^{-2}}\sqrt{\frac{1.16 \times 10^{11}}{7.23 \times 10^{3}}}$$

$$n = 770.29 \times 10^2 \text{ Hz} = \boxed{77.03 \text{ kHz}}$$

***Problem 1.14 :** A quartz crystal of thickness 0.001 metre is vibrating at resonance. Calculate the fundamental frequency given that Y for quartz is 7.9 × 10^{10} N/m^2 and ρ for quartz is 2650 kg/m^3.*

Data : $t = 0.001$ m, $\lambda = 7.9 \times 10^{10}$ N/m^2, $\rho = 2650$ kg/m^3

Solution :

$$f = \frac{1}{2t}\sqrt{\frac{\lambda}{\rho}}$$

$$= \frac{1}{2 \times 0.001}\sqrt{\frac{7.9 \times 10^{10}}{2650}}$$

$$= 2729.9 \text{ kHz}$$

$$= \boxed{2730 \text{ kHz}}$$

1.10 INTRODUCTION TO DIELECTRICS

- A **Dielectric** is an insulating material in which all the electrons are tightly bound to the nucleus of an atom. There are no free electrons available for the conduction of electricity. Thus the electrical conductivity of dielectrics is very less, ideally it is zero.
- The distinction between a dielectric material and an insulator lies in the application to which it is employed. The best examples of dielectrics are glass, polymer, mica, oil and paper.
- The insulating materials are used to prevent the electrical flow of electric current to undesired locations, whereas the dielectrics are used to store electrical energy.

1.11 DIELECTRIC PARAMETERS

1.11.1 Dielectric Constant [May 18]

- It is observed that the storing of capacity of a capacitor increases if the space between its plate is filled with dielectric material. If C_0 is the capacitance in vacuum and C the capacitance when the space is filled with a dielectric material then the **Dielectric Constant** of the material is

$$K = \frac{C}{C_0} \qquad \text{... (1.44)}$$

- Thus, the dielectric constant of a material is the ratio of the capacitance of a capacitor completely filled with that material to the capacitance of the same capacitor in vacuum.

In other words, the ratio of permittivity of medium to that of the vacuum is,

i.e. $$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \qquad \ldots (1.45)$$

where,
K_r = dielectric constant
ε_r = relative permittivity.
ε = permittivity of material
ε_0 = permittivity of vacuum

1.11.2 Electric Displacement [May 18]

- The **Electric Displacement** field in a material is defined as,

$$D = \varepsilon_0 E + P \qquad \ldots (1.46)$$

where, ε_0 is the permittivity of the free space or vacuum, E is the electric field and P is the polarization density of the electric material.

- The electric displacement field is a vector field having unit c/m^2.
- The electric displacement field is a vector field which describes the displacement effects of an electric field on the charges within a dielectric material, such as polarization charges or bound charges.
- In short it is the charge per unit area that would be displaced across a layer of conductor placed across an electric field.

1.11.3 Polarization

- When an electric field E is applied to a dielectric material consisting of positive and negative charges, the positive charges move opposite to the direction of the field while negative charges in the direction of the field.
- The displacement of the charges creates a local dipole in the dielectric. This process is known as **Dielectric Polarization**.

Fig. 1.15 shows the concept of polarization.

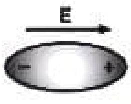


Fig. 1.15 : Polarization

- The polarization density is defined as induced dipole moment per unit volume.

1.11.4 Polarizability [May 18]

- When a dielectric material is placed in an electric field, the displacement of electric charges give rise to the creation of dipole in the material. The polarization P is directly proportional to the applied electric field E.

i.e. $$P \propto E$$

or $$P = \alpha E \qquad \ldots (1.47)$$

where α is a proportionality constant known as **Polarizability** and the unit is fm^2.

- If the dielectric material contains N dipoles per unit volume, then,

$$P = N \alpha E \qquad \ldots (1.48)$$

1.12 TYPES OF POLARIZATION

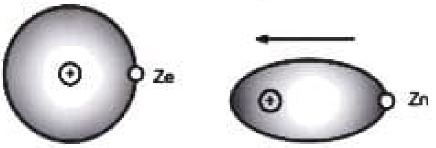
- When an electric field is applied to a dielectric material, it creates or realigns the dipoles resulting in polarization. The main types of polarization are categorised as below
  - ➢ Electronic polarization (P_e)
  - ➢ Ionic polarization (P_i)
  - ➢ Orientation polarization (P_o)

1.12.1 Electronic Polarization

- A dielectric material has large number of atoms having nuclei at the centre and electrons around it in different orbits. When an electric field is applied, the nucleus moves away from the field while the electrons towards the field. Therefore, there is a displacement which is less than the dimensions of the atom, the type of polarization is called **Electronic Polarization**.
- The electronic polarization is independent of temperature. The polarization is given by,

$$\bar{P}_e = N\alpha_e\bar{E} \qquad \ldots (1.49)$$

- The Fig. 1.16 shows electronic polarization.



(a) E = 0 (b) E = E

Fig. 1.16 : Electronic polarization

UNIT II

OPTICS, FIBRE OPTICS AND LASER

2.1 INTRODUCTION TO INTERFERENCE

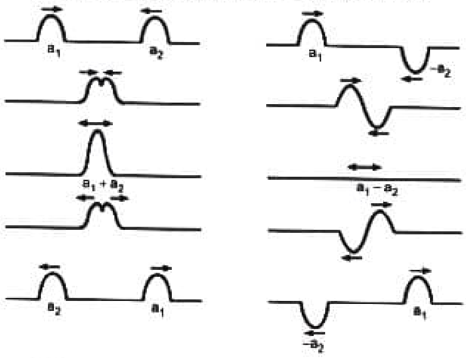
- The most common type of radiation which we come across in day to day life is electromagnetic wave or photon (the light quanta). Some of the electromagnetic waves can stimulate retina and some cannot. The part of the electromagnetic wave which can stimulate the retina is called **Light**.
- The branch of physics which deals with light is called **Optics**. Further, optics can be broadly classified as (a) **Geometrical Optics**, (b) **Physical or Wave Optics** and (c) **Quantum Optics** depending upon the basic behaviour of light assumed for explaining the optical phenomena. In the current course, our main focus will be on physical optics, where we assume the wave nature of light.
- From basic optical phenomena such as interference and diffraction, we can conclude that the light has a wave nature. In this unit, we will be studying these two basic properties of light.
- But these properties fail to explain the type of oscillations involved i.e. polarisation. The polarisation of light will be studied in later part of the text.
- The wave theory of light was proposed by Christian Huygen in 1679 but interference was demonstrated by Thomas Young only in 1802. There are several examples of interference that can be observed in everyday life. Basically, oil is colourless but a film of oil floating on water shows bright colours and also keeps on changing colour. Similarly a soap bubble, a compact disc, a thin sheet of mica or cellophane appear coloured. All this is due to interference of light.
- In engineering too, interference has wide applications such as measurement of thickness and stress, testing flatness of a surface, anti-reflecting coating etc.

2.1.1 Interference of Waves

- If two waves of same frequency travel in same direction with a constant phase difference with time, they combine so that their energy is not uniformly distributed in space, but is maximum at certain points and minimum (or zero) at other points. This phenomenon is called **Interference**.
- In interference, energy is neither created (at maxima) nor destroyed (at minima) but is redistributed so that there is more energy at certain points (maxima) and less energy at other points (minima). Even after interference the total energy of the system remains constant.
- Thus, interference is the redistribution of energy due to superposition of two or more waves.

Principle of Superposition

- The principle of superposition states that when two or more waves are superposed in space or a medium, the waves travel independently, through each other and the resultant displacement of each position is the algebraic/vector sum of the displacements due to each wave. Fig. 2.1 shows superposition of two waves.
- In Fig. 2.1 (a), two crests, with amplitude a_1 and a_2, are approaching each other and the point where they meet the resultant amplitude $(a_1 + a_2)$ is more than the individual amplitudes. After this, they pass through each other as though they have not interfered at all. Similarly, in Fig. 2.1 (b), one crest and one trough, with amplitude a_1 and $-a_2$, are approaching each other. At the point where they meet the resultant amplitude $(a_1 - a_2)$ is less than the individual amplitudes.
- The first case is called **Constructive Interference** and the second case is called **Destructive Interference**.



(a) Constructive Interference (b) Destructive interference

Fig. 2.1: Superposition

(1) Constructive Interference

- When the crest of one wave overlaps the crest of the other or the trough of one overlaps with the trough of the other, the displacement is maximum. This is called as **Constructive Interference**.

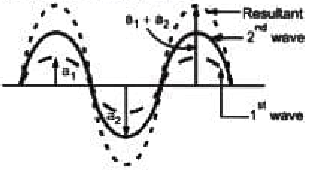


Fig. 2.2: Constructive interference

- In this case, the waves are in phase and the resultant amplitude equals the sum of the two component amplitudes.

i.e. $A = a_1 + a_2$ ... (2.1)

If $a_1 = a_2 = a$

then $A = 2a$ ... (2.2)

The resultant intensity will be

$$I = A^2 = 4a^2 \quad \text{... (2.3)}$$

- Here the path difference between two waves is 0. Constructive interference will also take place when the path difference is λ or 2λ. In general, condition for constructive interference is,

Path difference,

$$\Delta = n\lambda \quad \text{... (2.4)}$$

where $n = 0, 1, 2, \ldots n.$

In terms of phase difference,

Phase difference,

$$\delta = k\Delta, \text{ where } k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \delta = \frac{2\pi}{\lambda} \cdot n\lambda$$

$$\delta = 2n\pi \quad \text{... (2.5)}$$

(2) Destructive Interference

- In the other case, when the crest of one wave overlaps the trough of other or vice-versa, the displacement is minimum. This is called as **Destructive Interference**.

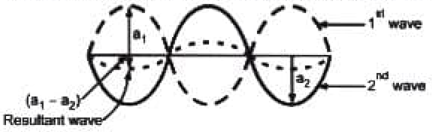


Fig. 2.3: Destructive interference

- In this case, the waves are 180° out of phase and the resultant amplitude is the difference of the two component amplitudes.

i.e. $A = a_1 - a_2$ ... (2.6)

If $a_1 = a_2 = a$

then $A = 0$ ... (2.7)

The resultant intensity,

$$I = A^2 = 0 \quad \text{... (2.8)}$$

- Here the path difference between two waves is λ/2. The same thing will happen when the path difference is 3λ/2 or 5λ/2. In general, destructive interference will occur when

Path difference,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda \quad \text{... (2.9)}$$

where $n = 0, 1, 2, \ldots, n.$

or Phase difference,

$$\delta = k \cdot \Delta \text{ where } k = \frac{2\pi}{\lambda}$$

$$\delta = \frac{2\pi}{\lambda}\left(n + \frac{1}{2}\right)\lambda$$

$$\delta = (2n + 1)\pi \quad \text{... (2.10)}$$

2.2 INTERFERENCE OF LIGHT IN THIN FILM

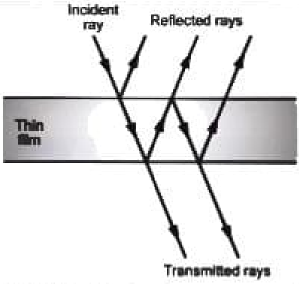


Fig. 2.4: Multiple reflections in a transparent thin film

- A film is said to be **Thin** when its thickness is of the order of wavelength of visible light (taken to be 5500 A°, which is the centre of the visible spectrum).
- A thin film may be a thin sheet of transparent material like glass, mica or an air film enclosed between two transparent sheets or a soap bubble.
- A light ray incident on a thin transparent film undergo reflection from the upper and lower surfaces of the film. They travel along different paths and may be reunited to produce **Interference**.

In a thin film, a small portion gets reflected from the upper surface while a major portion is transmitted into the film. The lower surface reflects a small portion of the transmitted component, back into the film while the rest of it emerges out of the film from other side.

Hence, a small portion of light gets partially reflected in succession several times within the film (as shown in Fig. 2.4). In a transparent thin film, the two surfaces strongly transmit and weakly reflect the incident light. In such cases, only the first few reflections at the top surface and the first few reflections at the bottom surface will be of appreciable strength. Hence, only the first two rays will be considered in the discussion.

At each reflection, the incident amplitude is divided into a reflected and transmitted component. Therefore, interference in thin films is called **Interference by Division of Amplitude**. This phenomenon was first observed by Newton and Robert Hooke but was correctly explained by Thomas Young.

In the Ongoing Discussion, Following Facts are Assumed

- When a ray of light gets **Reflected** from a **Denser Medium** into a **Rarer Medium**, it undergoes a **Phase Change of π** or a **Path Change of $\lambda/2$**.
- A **Distance 't'** traversed by light in a medium of **Refractive Index 'μ'** has an equivalent **Optical Path 'μ T'**.

.3 INTERFERENCE DUE TO THIN FILMS OF UNIFORM THICKNESS [Dec. 17]

Consider a thin film of uniform thickness 't' and refractive index 'μ'. Let XY and X'Y' be the faces of this parallel sided film. The film is surrounded by air on both the sides.

A plane monochromatic light ray, which can be considered as a parallel beam, is incident on the upper surface of the film.

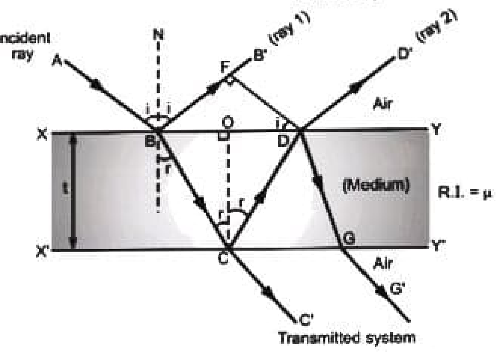


Fig. 2.5: Reflection of light from a parallel thin film

- Let AB represent one of the incident rays. The light ray travelling along AB is incident at an angle 'i' on the upper surface of the film. A part of the incident light is reflected at the upper face along BB' and a part is refracted at an angle 'r' along BC.
- At C, it is partly reflected back into the film along CD while a major portion is transmitted along CC'. The ray along CD emerges along DD' parallel to BB'.
- Both rays BB' (ray 1) and DD' (ray 2) are obtained from the same incident ray. They are, therefore, coherent and can produce interference. The condition of interference depends on the optical path difference between rays 1 and 2.
- To compute the optical path difference between the reflected ray BB' (ray 1) and the refracted ray DD' (ray 2), draw a normal DF from D on line BB'. Beyond points F and D, both rays travel equal distances. Hence will not be considered while calculating optical path difference.
- While ray BB' has covered a distance BF in air, the ray DD' has covered a distance BCD in the film of refractive index μ.
- The geometric **Path Difference** between ray 1 and ray 2 is, BC + CD − BF.

  ∴ Optical path difference is,

$$\Delta = \mu(BC + CD) - BF \qquad \dots (2.11)$$

(Since, distances BC and CD are travelled in medium of R.I. = μ)

From Δ BCO and Δ DCO, $\cos r = \dfrac{CO}{BC}$ and $\cos r = \dfrac{CO}{CD}$

$$\therefore \quad \cos r = \frac{t}{BC} \text{ and } \cos r = \frac{t}{CD} \quad (\because CO = t$$

$$\therefore \quad BC = CD = \frac{t}{\cos r} \qquad \dots (2.12)$$

and BF = BD sin i (from Δ BFD)

But BO = OD = BC sin r (from Δ BCO and Δ DCO)

∴ BF = 2 BC sin r sin i as BD = BO + OD

From equation (2.12), $BF = \dfrac{2t}{\cos r} \sin r \sin i$

Dividing and multiplying by sin r,

$$BF = \frac{2t}{\cos r} \sin^2 r \, \frac{\sin i}{\sin r}$$

$$BF = \frac{2\mu t}{\cos r} \sin^2 r \left(\because \frac{\sin i}{\sin r} = \mu\right) \dots (2.13)$$

Substituting equations (2.12) and (2.13) in equation (2.11),

$$\Delta = \mu\left(\frac{t}{\cos r}+\frac{t}{\cos r}\right)-\frac{2\mu t}{\cos r}\sin^2 r$$

$$\Delta = \frac{2\mu t}{\cos r}-\frac{2\mu t}{\cos r}\sin^2 r$$

$$\Delta = (1-\sin^2 r)\frac{2\mu t}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}\cos^2 r \quad (\because \sin^2 r+\cos^2 r = 1)$$

$$\Delta = 2\mu t\cos r \qquad \text{... (2.14)}$$

2.3.1 In Reflected System [Dec. 17]

- The path difference given by (2.14) is not the true optical path difference between rays 1 and 2. The phase change due to reflection is to be taken into account.
- At B, the reflection is in a rarer medium. So, a path change of $\lambda/2$ occurs in the reflected ray BB'. At C, reflection is in a denser medium, therefore no path change occurs in ray 2.

  $\therefore$ Total path difference between rays 1 and 2 is given by,

Total path difference = Path difference due to thin film + Path difference due to reflections

$$\Delta = 2\mu t\cos r \pm \frac{\lambda}{2} \qquad \text{... (2.15)}$$

(i) Condition for Constructive Interference

- If the total path difference is equal to an integral multiple of λ then rays 1 and 2 meet in phase and undergo constructive interference.

i.e., $\Delta = n\lambda$

$\therefore$ $2\mu t\cos r \pm \lambda/2 = n\lambda$

$2\mu t\cos r = (2n \pm 1)\,\lambda/2$ where $n = 0, 1, 2, 3 \ldots$ (2.16)

(ii) Condition for Destructive Interference

- If the optical path difference is equal to an odd integral multiple of $\lambda/2$, then rays 1 and 2 meet in opposite phase and undergo destructive interference.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$\therefore$ $2\mu t\cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$2\mu t\cos r = n\lambda$ where $n = 0, 1, 2$

... and is called order of interference ...(2.17)

- The rays incident on the film at the same angle are divided into two rays which become parallel on reflection from the surfaces of the film. Parallel rays do not intersect at finite distances, hence fringes are not observed at finite distances.
- The rays are to be condensed by a lens and interference is observed in its focal plane. Else, it can be observed by the unaided eye focused at infinity. Therefore, these interference fringes are said to be **Localized at Infinity**.

(iii) Important Cases

- If the film is extremely thin i.e. $t \ll \lambda$ or $t \to 0$ then the path difference, $\Delta = \lambda/2$. The film will appear dark in reflected light.
- When monochromatic light is incident normal to the film then $\cos r = 1$.

$$2\mu t = (2n+1)\frac{\lambda}{2} \text{ for brightness}$$

and $2\mu t = n\lambda$ for darkness.

- This implies that the film will appear bright in reflected light if the film has thickness of

$$t = \frac{\lambda}{4\mu}, \frac{3\lambda}{4\mu}, \ldots\ldots\ldots \text{ and it will appear dark}$$

for a thickness of $t = \frac{\lambda}{2\mu}, \frac{2\lambda}{2\mu}, \frac{3\lambda}{2\mu} \ldots\ldots$

- If the incident monochromatic light is parallel, the whole film will be uniformly bright or dark as film thickness 't' and angle of refraction 'r' are constant. For a given incident wavelength (say green) the condition of constructive interference causes the incident colour to intensify (intense green).
- A change in the angle of incidence of the rays causes a change in the path difference. The optical path difference decreases with increase in angle of incidence. Hence, as inclination of the film is changed, it will appear alternately dark and bright for incident monochromatic light.
- If white light is incident on the film, the optical path difference will vary from one colour to the other as λ is different. Hence, the film will appear coloured, the colour being that of the rays which interfered constructively. Further, as the inclination of the film is changed, the film will appear coloured.
- If the incident white light is not parallel, the optical path difference will change due to change in the incident angle. Hence, the film will show different colours when viewed from different directions.

2.3.2 In Transmitted System

- When the film is observed in transmitted light, it can be shown that the path difference between rays CC' and GG' (Fig. 2.5) is equal to $2\mu t \cos r$. Reflections at C and D are in a denser medium. So, no additional path change will occur due to reflection.

Total path difference = Path difference due to thin film
+ Path difference due to reflections

$$\therefore \quad \Delta = 2\mu t \cos r + 0 \qquad \text{... (2.18)}$$

(i) Condition for Constructive Interference

- For constructive interference the total phase difference should be an integral multiple of λ.

i.e. $\Delta = n\lambda$

$$\therefore \quad 2\mu t \cos r = n\lambda \qquad \text{... (2.19)}$$

(ii) Condition for Destructive Interference

- For destructive interference the total phase difference should be an odd integral multiple of $\lambda/2$.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \qquad \text{... (2.20)}$$

As is evident, the condition for brightness on reflection becomes the condition for darkness on transmission and vice versa.

2.4 INTERFERENCE IN FILMS OF NON-UNIFORM THICKNESS (WEDGE SHAPED FILM)

- A **Wedge** is a plate or film of varying thickness, having zero thickness at one end and progressively increasing to a particular thickness at the other end.
- Consider two plane surfaces XY and X'Y' inclined at an angle α. The thickness of the film increases linearly from X to Y.
- When the wedge is illuminated by a parallel beam of monochromatic light, the rays reflected from its two surfaces will not be parallel. They appear to diverge from a point S near the film.
- When the film is viewed with reflected monochromatic light, **Equidistant Interference Fringes** are observed which are parallel to the line of intersection of the two surfaces. The fringes are alternately bright and dark and are localised at the surface of the film.
- On illuminating the film with monochromatic light, one system of rays is reflected from the front surface XY and the other system of rays is obtained by transmission at the back surface X'Y' (not shown in Fig. 2.6) and consequent reflections at the front surface. As both rays are obtained from a single source, they are coherent and produce interference.

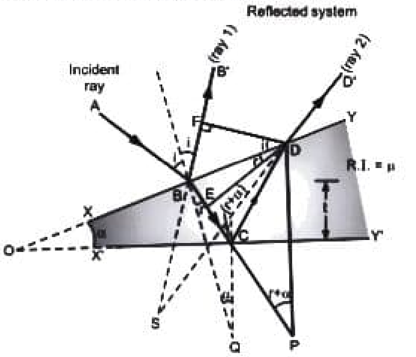


Fig. 2.6: Wedge-shaped film

The interfering rays BB' and DD' are not parallel but appear to diverge from a point S. The optical path difference between them is given by,

$$\Delta = \mu(BC + CD) - BF$$

By the definition of optical path difference,

$$BF = \mu(BE)$$

$$\therefore \quad \Delta = \mu(BE + EC + CD) - \mu(BE)$$

$$\therefore \quad \Delta = \mu(EC + CD)$$

$$\Delta = \mu(EC + CP)$$

($\because$ CPD is an isosceles triangle)

$$\therefore \quad \Delta = \mu(EP)$$

$$\therefore \quad \Delta = 2\mu t \cos(r + \alpha) [\text{as } EP = DP \cos(r + \alpha)$$

$$= 2t \cos(r + \alpha)$$

In Reflected System:

Due to reflection, an additional path change is introduced in the reflected system at point B.

$\therefore$ Total path difference = Path difference due to thin film
+ Path difference due to reflections

$$\Delta = 2\mu t \cos(r + \alpha) \pm \frac{\lambda}{2} \qquad \text{... (2.21)}$$

Condition for Constructive Interference:

For constructive interference the total phase difference should be an integral multiple of λ.

$$\Delta = n\lambda$$

$$\text{i.e. } 2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = n\lambda$$

$$\therefore \quad 2\mu t \cos(r+\alpha) = (2n \pm 1)\frac{\lambda}{2} \qquad \text{... (2.22)}$$

Condition for Destructive Interference:

For destructive interference the total phase difference should be an odd integral multiple of λ/2.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\text{i.e. } 2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \quad 2\mu t \cos(r+\alpha) = n\lambda \qquad \text{... (2.23)}$$

Nature of Interference Pattern :

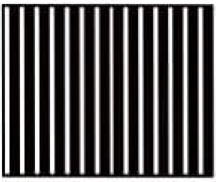


Fig. 2.7: Interference pattern in wedge-shaped film. Alternately bright and dark bands are parallel

If the film is illuminated by parallel light, 'i' is constant everywhere and so is 'r', the angle of refraction. In addition, if monochromatic light is used, the path change will occur only due to 't'. In this case, the fringes will be of **Equal Thickness**. For a wedge shaped film, 't' remains constant only in a direction parallel to the thin edge of the wedge. So, straight fringes parallel to the edge of the wedge are obtained. The fringes are alternately bright or dark for monochromatic light. For white light, coloured fringes are obtained.

[**Note** : In transmitted system we will get exactly opposite of the reflected system.]

2.5 FRINGE WIDTH (β)

- When a wedge film is illuminated by monochromatic light of wavelength λ, it gives fringes of equal thickness. Fringe width can be calculated by knowing the position of consecutive minima or maxima. Here, for mathematical simplicity, we will consider minima.

For n^{th} minimum, we have

$$2\mu t \cos(r+\alpha) = n\lambda$$

For normal incidence,

$$r = 0$$

$$\therefore \quad 2t\mu \cos\alpha = n\lambda \qquad \text{... (2.24)}$$

Let this n^{th} dark band be formed at a distance x_n from the thin edge.

$$\therefore \quad t = x_n \tan\alpha \qquad \text{(from Fig. 2.8) ... (2.25)}$$

From equations (2.24) and (2.25),

$$2\mu x_n \tan\alpha \cos\alpha = n\lambda$$

$$\text{or} \quad 2\mu x_n \sin\alpha = n\lambda \qquad \text{... (2.26)}$$

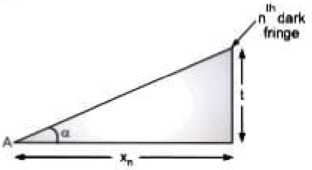


Fig. 2.8

- Similarly, if the $(n+1)^{th}$ minimum is obtained at a distance x_{n+1} from the thin edge, then

$$2\mu x_{n+1} \sin\alpha = (n+1)\lambda \qquad \text{... (2.27)}$$

- Subtracting equation (2.26) from equation (2.27), we get fringe width of a bright fringe.

$$2\mu (x_{n+1} - x_n) \sin\alpha = \lambda$$

∴ Fringe width,

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\mu \sin\alpha}$$

$$\beta = \frac{\lambda}{2\mu \sin\alpha}$$

$$\cong \frac{\lambda}{2\mu\alpha} \quad \text{(for small } \alpha \text{ and in radians) ... (2.28)}$$

- For an air film ($\mu = 1$), fringe width,

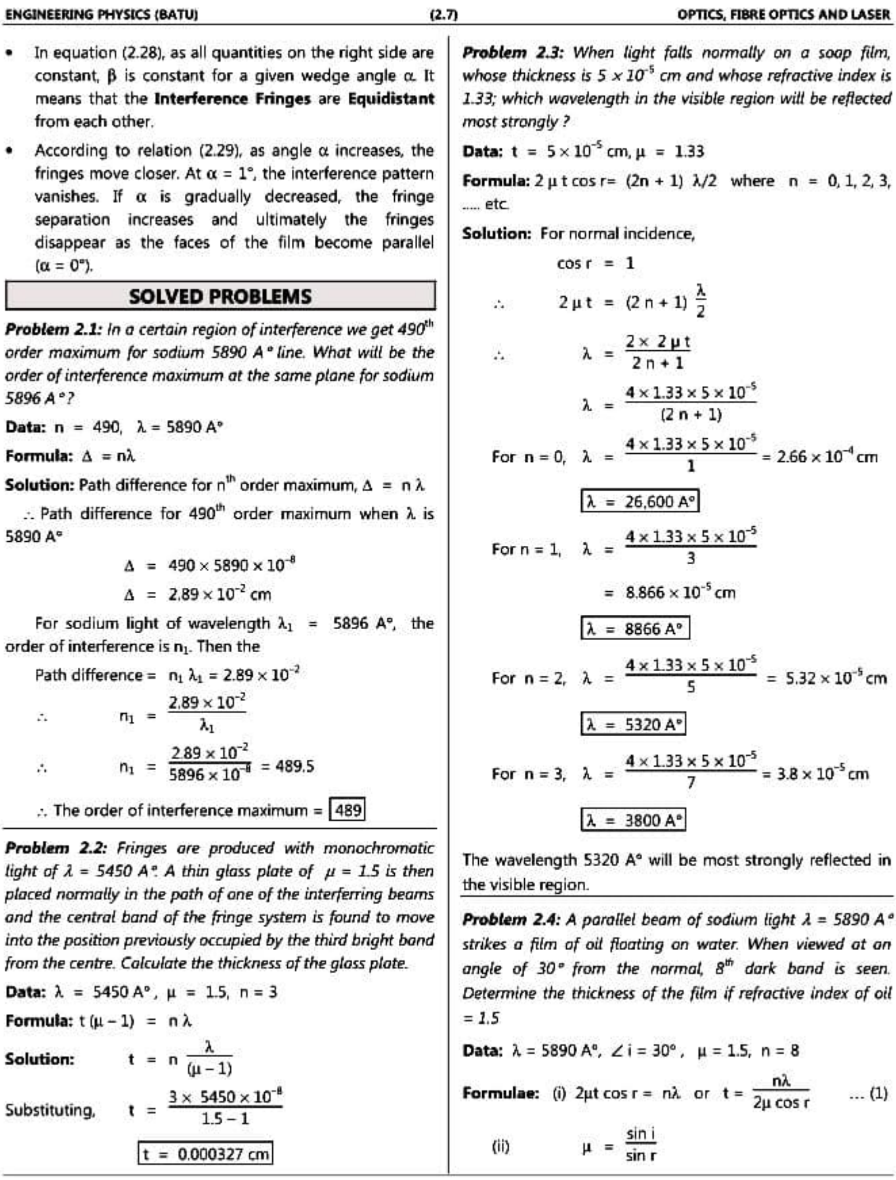
$$\beta = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{2\alpha} \text{ (for small } \alpha) \qquad \text{... (2.29)}$$

- Similarly, it can be shown that the fringe width for dark fringes is given by,

$$\beta = \frac{\lambda}{2\mu \sin\alpha} \qquad \text{... (2.30)}$$

which is the same as that of a bright fringe.

- The width of a dark or bright fringe is however equal to half the fringe width.



- In equation (2.28), as all quantities on the right side are constant, β is constant for a given wedge angle α. It means that the **Interference Fringes** are **Equidistant** from each other.
- According to relation (2.29), as angle α increases, the fringes move closer. At $\alpha = 1°$, the interference pattern vanishes. If α is gradually decreased, the fringe separation increases and ultimately the fringes disappear as the faces of the film become parallel ($\alpha = 0°$).

SOLVED PROBLEMS

Problem 2.1: *In a certain region of interference we get 490th order maximum for sodium 5890 A° line. What will be the order of interference maximum at the same plane for sodium 5896 A°?*

Data: $n = 490, \ \lambda = 5890$ A°

Formula: $\Delta = n\lambda$

Solution: Path difference for nth order maximum, $\Delta = n\lambda$

$\therefore$ Path difference for 490th order maximum when λ is 5890 A°

$$\Delta = 490 \times 5890 \times 10^{-8}$$

$$\Delta = 2.89 \times 10^{-2} \text{ cm}$$

For sodium light of wavelength $\lambda_1 = 5896$ A°, the order of interference is n_1. Then the

$$\text{Path difference} = n_1 \lambda_1 = 2.89 \times 10^{-2}$$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{\lambda_1}$$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{5896 \times 10^{-8}} = 489.5$$

$\therefore$ The order of interference maximum = $\boxed{489}$

Problem 2.2: *Fringes are produced with monochromatic light of $\lambda = 5450$ A°. A thin glass plate of $\mu = 1.5$ is then placed normally in the path of one of the interferring beams and the central band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. Calculate the thickness of the glass plate.*

Data: $\lambda = 5450$ A°, $\mu = 1.5, \ n = 3$

Formula: $t(\mu - 1) = n\lambda$

Solution: $$t = n \frac{\lambda}{(\mu - 1)}$$

Substituting, $$t = \frac{3 \times 5450 \times 10^{-8}}{1.5 - 1}$$

$$\boxed{t = 0.000327 \text{ cm}}$$

Problem 2.3: *When light falls normally on a soap film, whose thickness is 5×10^{-5} cm and whose refractive index is 1.33; which wavelength in the visible region will be reflected most strongly?*

Data: $t = 5 \times 10^{-5}$ cm, $\mu = 1.33$

Formula: $2\mu t \cos r = (2n + 1)\lambda/2$ where $n = 0, 1, 2, 3,$ ..... etc.

Solution: For normal incidence,

$$\cos r = 1$$

$$\therefore \quad 2\mu t = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2 \times 2\mu t}{2n + 1}$$

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n + 1)}$$

For $n = 0$, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1} = 2.66 \times 10^{-4} \text{ cm}$$

$$\boxed{\lambda = 26{,}600 \text{ A°}}$$

For $n = 1$, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{3}$$

$$= 8.866 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 8866 \text{ A°}}$$

For $n = 2$, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{5} = 5.32 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 5320 \text{ A°}}$$

For $n = 3$, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{7} = 3.8 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 3800 \text{ A°}}$$

The wavelength 5320 A° will be most strongly reflected in the visible region.

Problem 2.4: *A parallel beam of sodium light $\lambda = 5890$ A° strikes a film of oil floating on water. When viewed at an angle of 30° from the normal, 8th dark band is seen. Determine the thickness of the film if refractive index of oil = 1.5*

Data: $\lambda = 5890$ A°, $\angle i = 30°$, $\mu = 1.5, \ n = 8$

Formulae: (i) $2\mu t \cos r = n\lambda$ or $t = \dfrac{n\lambda}{2\mu \cos r}$ ... (1)

(ii) $$\mu = \frac{\sin i}{\sin r}$$

Solution: $\sin r = \dfrac{\sin i}{\mu}$

$$\cos r = \sqrt{1-\sin^2 r}$$

$$\therefore \quad \cos r = \sqrt{1-\frac{\sin^2 i}{\mu^2}}$$

$$\therefore \quad \cos r = \sqrt{1-\frac{\sin^2 30}{(1.5)^2}}$$

$$\cos r = 0.943$$

Substituting in (1),

$$\therefore \quad t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.943}$$

$$= \boxed{1.6302 \times 10^{-4}\ \text{cm}}$$

Problem 2.5: *Two glass plates enclose a wedge-shaped air film, touching at one edge and are separated by a wire of 0.03 mm diameter at a distance of 15 cm from the edge. Monochromatic light of λ = 6000 A° from a broad source falls normally on the film. Calculate the fringe width of the fringes thus formed.*

Data: $\lambda = 6000 \times 10^{-8}$ cm; For air film, $\mu = 1$

Diameter = 0.03 mm = 0.003 cm

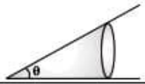
Distance of fringe from the edge = 15 cm

Formula: Fringe width,

$$\beta = \frac{\lambda}{2\mu \sin \alpha} = \frac{\lambda}{2\mu \tan \alpha}$$

Solution:

$$\beta = \frac{6000 \times 10^{-8}}{2 \times 1 \times \frac{0.003}{15}} = \boxed{0.15\ \text{cm}}$$



Problem 2.6: *Interference fringes are produced by monochromatic light falling normally on a wedge-shaped film of cellophane whose refractive index is 1.4. The angle of the wedge is 20 sec of an arc and the distance between the successive fringes is 0.25 cm. Calculate the wavelength of light.*

Data: $\beta = 0.25$ cm, $\mu = 1.4$, $\theta = 20$ sec

$$= \frac{20}{60 \times 60} \times \frac{\pi}{180}$$

$$= \frac{1}{180} \times \frac{\pi}{180} \text{ radians}$$

Formula: $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\therefore \quad \lambda = 2\mu\, \alpha \cdot \beta$$

Solution: $\lambda = 2 \times 1.4 \times \dfrac{\pi}{180 \times 180} \times 0.25$

$$\boxed{\lambda = 6.79 \times 10^{-5}\ \text{cm}}$$

Problem 2.7: *Two plane rectangular pieces of glass are in contact at one edge and separated by a hair at opposite edge, so that a wedge is formed. When light of wavelength 6000 A° falls normally on the wedge, nine interference fringes are observed. What is the thickness of the hair ?*

Data: $\lambda = 6000 \times 10^{-8}$ cm, $n = 9$, $r = 0$ for normal incidence

Formula: $2\mu t \cos (r + \alpha) = n\lambda$

Solution: If the fringes are seen normally and the angle of wedge is very small, then r = 0, so that

$$\cos (r + \alpha) = \cos \alpha = 1$$

For air film, $\mu = 1$

$$\therefore \quad 2\mu t = n\lambda$$

$$2 \times 1 \times t = 9 \times 6000 \times 10^{-8}$$

$$\therefore \quad t = \frac{9 \times 6000 \times 10^{-8}}{2} = \boxed{27 \times 10^{-5}\ \text{cm}}$$

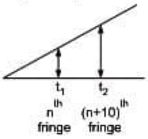
Problem 2.8: *A square piece of cellophane film with index of refraction 1.5 has a wedge-shaped section, so that its thickness at two opposite sides is t_1 and t_2. If with light of λ = 6000 A°, the number of fringes appearing on the film is 10, calculate the difference $t_2 - t_1$.*

Data: $\lambda = 6000 \times 10^{-8}$ cm, $\mu = 1.5$

Formula: $2\mu t_1 \cos (r + \alpha) = \lambda$ ... (1)

Solution: For $(n + 10)^{th}$ dark fringe,

$2\mu t_2 \cos (r + \alpha) = (n + 10)\lambda$ ... (2)



For normal incidence, r = 0 and if the angle of wedge is small,

$$\cos (r + \alpha) = \cos \alpha = 1$$

∴ Equations (1) and (2) become

$2\mu t_1 = n\lambda$ ... (3)

$2\mu t_2 = (n + 10)\lambda$ ... (4)

∴ Subtracting equation (3) from equation (4), we get

$$2\,\mu\,(t_2 - t_1) = 10\,\lambda$$

$$\therefore \quad t_2 - t_1 = \frac{10\,\lambda}{2\,\mu} = \frac{10 \times 6000 \times 10^{-8}}{2 \times 1.5}$$

i.e. $\boxed{t_2 - t_1 = 2 \times 10^{-4}\ \text{cm}}$

Problem 2.9: *A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.*

Data: $\lambda = 5890$ A°, $r = 60°$, $\mu = 1.5$

Formula: For darkness,

$$2\mu t \cos r = n\lambda. \quad \text{Let } n = 1$$

Solution:

$$t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$

$$\boxed{t = 3.926 \times 10^{-5}\ \text{cm}}$$

Problem 2.10: *The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used.*

Data: $\Delta_1 = 199.5\,\lambda$, $\Delta_2 = 0.012$ cm

Formula: For 199.5 λ it is odd path difference, therefore point is a dark fringe and for darkness,

$$\Delta = (2n-1)\frac{\lambda}{2} \qquad \left(\because \left(\frac{2n-1}{2}\right) = 199.5\right)$$

Solution: $0.012 = 199.5\,\lambda$

$$\therefore \quad \lambda = \frac{0.012}{199.5}$$

$$\therefore \quad \lambda = 6.015 \times 10^{-8}\ \text{cm}$$

$$\boxed{\lambda = 6015\ \text{A°}}$$

Problem 2.11: *Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm.*

Solution: 18 bands per cm

$$\therefore \text{ Band width, } \beta = \frac{1}{18}$$

$$\beta = 0.0556\ \text{cm}$$

We know, $\beta = \dfrac{\lambda}{2\alpha}$

$$\alpha = \frac{\lambda}{2\beta} = \frac{4800 \times 10^{-8}}{2 \times 0.0556}$$

$$\alpha = 4.3165 \times 10^{-4}\ \text{rad}$$

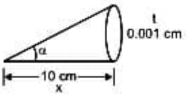
Note conversion of radians into seconds.

$$\alpha = 4.3165 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60$$

$$\boxed{\alpha = 89.02\ \text{seconds}}$$

Problem 2.12: *Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes.*

Solution: $\tan\alpha = \dfrac{t}{x}$



As α is very small, tan α = α

$$\alpha = \frac{t}{x} = \frac{0.001}{10}$$

$$\alpha = 0.0001\ \text{rad}$$

$$\beta = \frac{\lambda}{2\alpha} = \frac{5.9 \times 10^{-5}}{2 \times 0.0001}$$

$$\boxed{\beta = 0.295\ \text{cm}}$$

Note while using α in calculation it must be all the time in radians. If it is given in seconds then convert it in rad.

Problem 2.13: *Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°.*

Data: $n = 4$, $\lambda = 5893$ A°

Formula: For bright band and wedge-shaped film,

$$2\mu t \cos(r + \alpha) = (2n-1)\frac{\lambda}{2}$$

Let normal incidence, $r = 0$ and α is very small

$$\therefore \quad \cos(r+\alpha) = 1,\ \mu = 1$$

$$\therefore \quad 2t = (2n-1)\frac{\lambda}{2}$$

Solution:

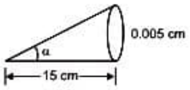
$$2t = \frac{(2 \times 4 - 1)\,\lambda}{2}$$

$$t = \frac{7}{4} \times 5893 \times 10^{-8}$$

$$\boxed{t = 1.031275 \times 10^{-4}\ \text{cm}}$$

***Problem 2.14:** Monochromatic light emitted by a broad source of light of wavelength 6×10^{-5} cm falls normally on two glass plates which enclose a thin wedge-shaped film of air. The plates touch at one end and are separated at a point 15 cm from the end by a wire 0.5 mm in diameter. Find the width between any two consecutive bright fringes.*

Solution:



$$\alpha = \frac{t}{x} = \frac{0.005}{15} = 3.333 \times 10^{-4} \text{ rad}$$

$$\text{Bandwidth, } \beta = \frac{\lambda}{2\alpha}$$

$$\beta = \frac{6 \times 10^{-5}}{2 \times 3.333 \times 10^{-4}}$$

$$\boxed{\beta = 0.09 \text{ cm}}$$

***Problem 2.15:** Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 A°. Calculate the angle of wedge in seconds of an arc.*

Data:

$\mu = 1.52$

$\lambda = 5893$ A°

$\beta = 1$ mm

Formula: Fringe width is,

$$\beta = \frac{\lambda}{2\alpha}$$

Solution: $1 \times 10^{-1} = \dfrac{5893 \times 10^{-8}}{2\alpha}$

$$\therefore \quad \alpha = \frac{5893 \times 10^{-8}}{2 \times 1 \times 10^{-1}}$$

$$\alpha = 2.9 \times 10^{-4} \text{ radian}$$

$$\text{or} \quad \alpha = \frac{2.9 \times 10^{-4} \times 180 \times 60 \times 60}{3.14} \text{ of an arc}$$

$$= \boxed{59.8 \text{ sec.}}$$

***Problem 2.16:** A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, in the reflected light, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 A°.*

Data:

$i = 30°$

$\mu = 1.46$

$\lambda = 5890$ A°

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\dfrac{\sin 30}{\sin r} = 1.46$

$$\sin r = \frac{0.5}{1.46} = 0.34247$$

$$\therefore \quad \boxed{r = 20°}$$

(iii) The thickness is given by relation (condition for minima)

$$2\mu t \cos r = n\lambda$$

$$t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times \cos 20°}$$

$$\boxed{t = 1.7 \times 10^{-4} \text{ cm}}$$

***Problem 2.17:** A soap film of refractive index 4/3 and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark and corresponding to wavelength of 5×10^{-5} cm. Calculate the order of interference band.*

Data:

$\mu = 4/3 = 1.33$

$t = 1.5 \times 10^{-4}$ cm

$i = 45°$

$\lambda = 5 \times 10^{-5}$ cm

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\dfrac{\sin 45°}{\sin r} = 1.33$

$$\sin r = \frac{0.707}{1.33}$$

$$\sin r = 0.53038$$

$$\text{or} \quad \boxed{r = 32°}$$

(ii) The order of interference will be given by (condition for dark band)

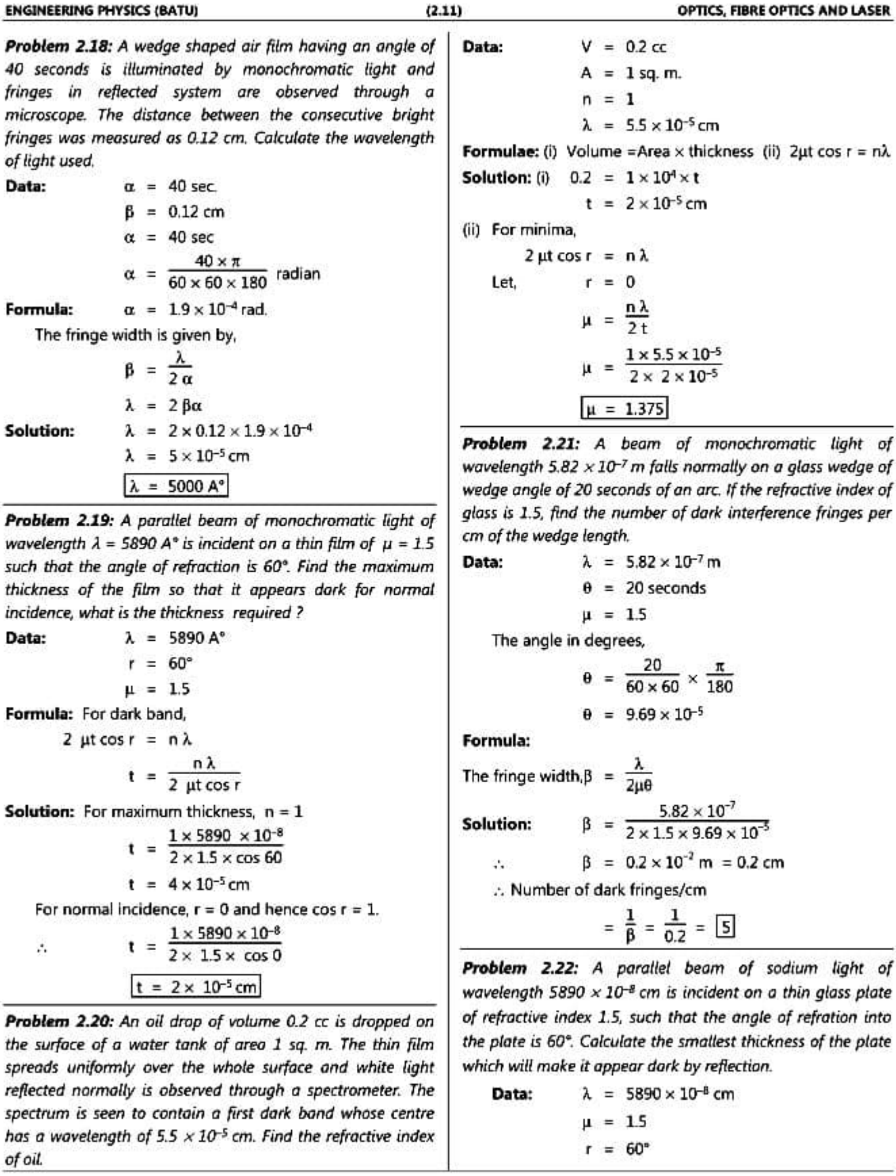
$$2\mu t \cos r = n\lambda$$

$$n = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times \cos 32°}{5 \times 10^{-5}}$$

$$\boxed{n = 6.78}$$

The order of interference,

$$n = 6$$



Problem 2.18: *A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes in reflected system are observed through a microscope. The distance between the consecutive bright fringes was measured as 0.12 cm. Calculate the wavelength of light used.*

Data: $\alpha = 40$ sec.

$\beta = 0.12$ cm

$\alpha = 40$ sec

$$\alpha = \frac{40 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

Formula: $\alpha = 1.9 \times 10^{-4}$ rad.

The fringe width is given by,

$$\beta = \frac{\lambda}{2\alpha}$$

$$\lambda = 2\beta\alpha$$

Solution: $\lambda = 2 \times 0.12 \times 1.9 \times 10^{-4}$

$\lambda = 5 \times 10^{-5}$ cm

$$\boxed{\lambda = 5000 \text{ A}^\circ}$$

Problem 2.19: *A parallel beam of monochromatic light of wavelength $\lambda = 5890$ A° is incident on a thin film of $\mu = 1.5$ such that the angle of refraction is 60°. Find the maximum thickness of the film so that it appears dark for normal incidence, what is the thickness required ?*

Data: $\lambda = 5890$ A°

$r = 60^\circ$

$\mu = 1.5$

Formula: For dark band,

$$2\ \mu t \cos r = n\lambda$$

$$t = \frac{n\lambda}{2\ \mu t \cos r}$$

Solution: For maximum thickness, n = 1

$$t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$

$t = 4 \times 10^{-5}$ cm

For normal incidence, r = 0 and hence cos r = 1.

$$\therefore \quad t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 0}$$

$$\boxed{t = 2 \times 10^{-5} \text{ cm}}$$

Problem 2.20: *An oil drop of volume 0.2 cc is dropped on the surface of a water tank of area 1 sq. m. The thin film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain a first dark band whose centre has a wavelength of 5.5×10^{-5} cm. Find the refractive index of oil.*

Data: $V = 0.2$ cc

$A = 1$ sq. m.

$n = 1$

$\lambda = 5.5 \times 10^{-5}$ cm

Formulae: (i) Volume = Area × thickness (ii) $2\mu t \cos r = n\lambda$

Solution: (i) $0.2 = 1 \times 10^{4} \times t$

$t = 2 \times 10^{-5}$ cm

(ii) For minima,

$$2\ \mu t \cos r = n\lambda$$

Let, $r = 0$

$$\mu = \frac{n\lambda}{2t}$$

$$\mu = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5}}$$

$$\boxed{\mu = 1.375}$$

Problem 2.21: *A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge of wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length.*

Data: $\lambda = 5.82 \times 10^{-7}$ m

$\theta = 20$ seconds

$\mu = 1.5$

The angle in degrees,

$$\theta = \frac{20}{60 \times 60} \times \frac{\pi}{180}$$

$$\theta = 9.69 \times 10^{-5}$$

Formula:

The fringe width, $\beta = \dfrac{\lambda}{2\mu\theta}$

Solution: $$\beta = \frac{5.82 \times 10^{-7}}{2 \times 1.5 \times 9.69 \times 10^{-5}}$$

$\therefore \quad \beta = 0.2 \times 10^{-2}$ m $= 0.2$ cm

$\therefore$ Number of dark fringes/cm

$$= \frac{1}{\beta} = \frac{1}{0.2} = \boxed{5}$$

Problem 2.22: *A parallel beam of sodium light of wavelength 5890×10^{-8} cm is incident on a thin glass plate of refractive index 1.5, such that the angle of refration into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.*

Data: $\lambda = 5890 \times 10^{-8}$ cm

$\mu = 1.5$

$r = 60^\circ$

Formula: The condition for dark fringe in reflected system is

$$2\mu t \cos r = n\lambda$$

Solution: Taking $n = 1$

$$2 \times 1.5 \times t \times \cos 60 = 5890 \times 10^{-8}$$

$$\therefore \quad t = 3.926 \times 10^{-3} \text{ cm}$$

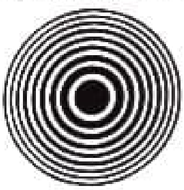
$$= \boxed{3.926 \times 10^{-5} \text{cm}}$$

2.6 NEWTON'S RINGS [May 18, 19]

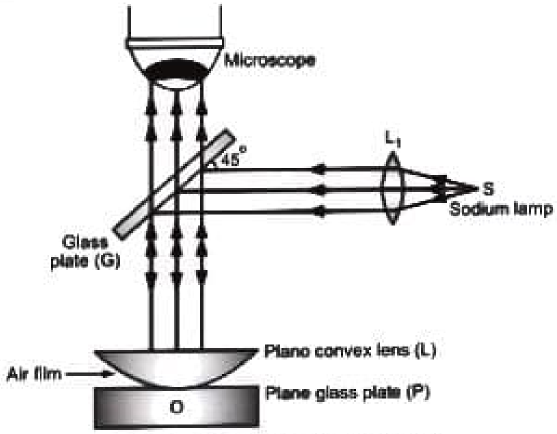
- When a **Plano-Convex Lens** of large focal length with its convex surface is placed in contact with a **Plane Glass Plate**, an air film of gradually increasing thickness is formed between them. The thickness of the film at the point of contact is zero and increases gradually outwards.
- If monochromatic light is allowed to fall normally, and the film is viewed in reflected light, alternate **Bright and Dark Rings** are observed. These rings are concentric around the point of contact between the lens and the glass plate. These fringes are called as **Newton's Rings** as they were discovered by Newton.

(i) Experimental Arrangement

- A plano-convex lens L of large radius of curvature is placed on a plane glass plate P. The point of contact between them is O. The light from an extended monochromatic source (sodium lamp) falls on a glass plate G held at an angle of 45° with the vertical.
- The glass plate G reflects normally a part of the incident light towards the air film between the lens L and the glass plate P. A part of the incident light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of plate P (i.e. rays are reflected from the top and bottom surfaces of the air film). These two reflected rays interfere and produce an **Interference Pattern** in the form of **Circular Rings**.
- These rings are **Localised** in the air film and can be seen with a microscope focused on the film.



(a) Typical Newton's rings pattern observed in reflected light



(b) Experimental arrangement for observing Newton's rings

Fig. 2.9

(ii) Explanation of the Formation of Newton's Rings

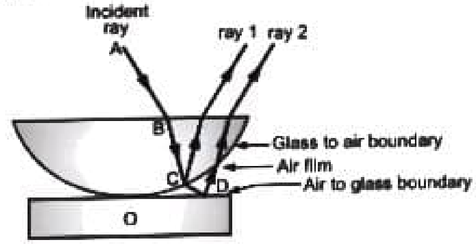


Fig. 2.10: Formation of Newton's rings

- When a monochromatic ray of light AB, is incident on the system, it gets partially reflected at C, the bottom of curved surface of the lens (glass-air boundary). This goes out in the form of ray 1 without any phase reversal. The other part is refracted along CD.
- At D, the top surface of the plane glass plate, it gets partially reflected to form ray 2. This ray has a phase reversal as it is reflected from air to glass boundary.
- As the rays 1 and 2 are derived from the same source and are coherent, so they interfere to form fringes. Interference does not take place between rays reflected from the surfaces of lens and glass plate due to their thickness which is much larger than wavelength of light.

(iii) Derivation

- The radius of curvature of plano-convex lens is very large and the small section of the air film trapped between lens and the glass plate will be similar to a wedged air film. Therefore, the optical path difference will be same as that of wedged air film.

The optical path difference for wedge film is,

$$\Delta = 2\mu t \cos(r + \alpha) \qquad \text{... (2.31)}$$

For air film, $\mu = 1$, for normal incidence

$$\cos r = 1 \text{ and } \alpha \approx 0$$

$$\therefore \quad \Delta = 2t \qquad \text{... (2.32)}$$

In Reflected System

Total optical path difference = Path difference due to thin film + Path difference due to reflections

$$\therefore \quad \Delta = 2t \pm \frac{\lambda}{2} \qquad \text{... (2.33)}$$

Condition for Constructive Interference

For constructive interference the total phase difference should be an integral multiple of λ.

$$\Delta = n\lambda$$

$$\therefore \quad 2t \pm \frac{\lambda}{2} = n\lambda$$

$$2t = \left(n \pm \frac{1}{2}\right)\lambda \qquad \text{... (2.34)}$$

Condition for Destructive Interference

For destructive interference the total phase difference should be an odd integral multiple of λ/2.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \quad 2t \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2t = n\lambda \qquad \text{... (2.35)}$$

Radii of Bright Rings

- The plano-convex lens LOL' is placed on a glass plate AB. The point C is the centre of the sphere of which LOL' is a part. Let R be the radius of curvature of the lens and r_n be the radius of the nth Newton's rings corresponding to the constant film thickness 't'.

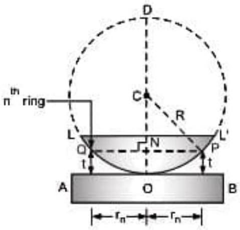


Fig. 2.11

- By the property of circle (theorem of intersecting chords),

$$NP \times NQ = NO \times ND$$

$$\text{i.e.} \quad r_n \times r_n = t(2R - t) = 2Rt - t^2 \approx 2Rt$$

(as t^2 is very small)

$$\therefore \quad r_n^2 = 2Rt \qquad \text{... (2.36)}$$

$$\text{or} \quad t = \frac{r_n^2}{2R} = \frac{D_n^2}{8R}$$

(D_n being diameter of nth bright ring) ... (2.37)

From equations (2.37) and (2.34), $\frac{2r_n^2}{2R} = (2n \pm 1)\frac{\lambda}{2}$

$$2 \cdot \frac{D_n^2}{8R} = (2n \pm 1)\frac{\lambda}{2}$$

$$D_n^2 = (2n \pm 1) \cdot 2\lambda R$$

$$D_n = \sqrt{2\lambda R} \cdot \sqrt{2n \pm 1} \text{ i.e. } D_n \propto \sqrt{2n \pm 1} \qquad \text{... (2.38)}$$

Equation (2.38) shows that diameter of a bright ring is proportional to the square root of odd natural numbers.

Radii of Dark Rings **[Dec. 18]**

The condition for formation of dark Newton's ring is,

$$2t = n\lambda \qquad \text{... (2.39)}$$

Substituting for t from (8),

$$2 \cdot \frac{D_n^2}{8R} = n\lambda$$

$$D_n^2 = 4n\lambda R \qquad \text{... (2.40)}$$

$$D_n = 2\sqrt{n\lambda R} \text{ i.e. } D_n \propto \sqrt{n} \qquad \text{... (2.41)}$$

Thus, the diameter of a dark ring is proportional to the square root of a natural number.

2.6.1 Properties of Newton's Rings

Rings get Closer Away from the Centre: Consider equation (2.41) giving the diameter of a dark ring. We have

$$D_n \propto \sqrt{n} \text{ and } D_{n+1} \propto \sqrt{n+1}$$

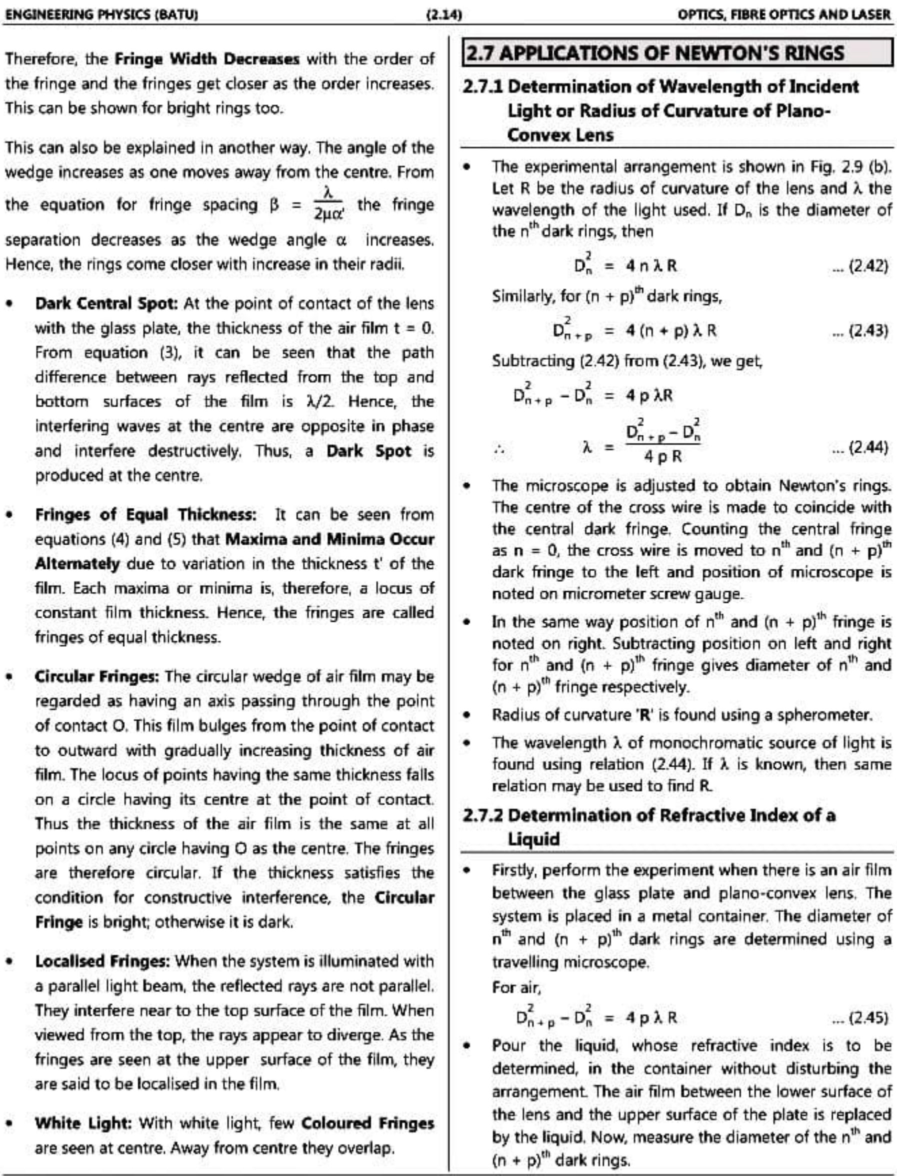
$$\therefore \quad D_{n+1} - D_n \propto (\sqrt{n+1} - \sqrt{n})$$

If constant of proportionality is taken as 1, then

$$D_{n+1} - D_n = \sqrt{n+1} - \sqrt{n}$$

$$\therefore \quad D_2 - D_1 = \sqrt{2} - \sqrt{1} = 0.414$$

$$D_3 - D_2 = \sqrt{3} - \sqrt{2} = 0.317$$



Therefore, the **Fringe Width Decreases** with the order of the fringe and the fringes get closer as the order increases. This can be shown for bright rings too.

This can also be explained in another way. The angle of the wedge increases as one moves away from the centre. From the equation for fringe spacing $\beta = \frac{\lambda}{2\mu\alpha'}$ the fringe separation decreases as the wedge angle α increases. Hence, the rings come closer with increase in their radii.

- **Dark Central Spot:** At the point of contact of the lens with the glass plate, the thickness of the air film t = 0. From equation (3), it can be seen that the path difference between rays reflected from the top and bottom surfaces of the film is $\lambda/2$. Hence, the interfering waves at the centre are opposite in phase and interfere destructively. Thus, a **Dark Spot** is produced at the centre.
- **Fringes of Equal Thickness:** It can be seen from equations (4) and (5) that **Maxima and Minima Occur Alternately** due to variation in the thickness t' of the film. Each maxima or minima is, therefore, a locus of constant film thickness. Hence, the fringes are called fringes of equal thickness.
- **Circular Fringes:** The circular wedge of air film may be regarded as having an axis passing through the point of contact O. This film bulges from the point of contact to outward with gradually increasing thickness of air film. The locus of points having the same thickness falls on a circle having its centre at the point of contact. Thus the thickness of the air film is the same at all points on any circle having O as the centre. The fringes are therefore circular. If the thickness satisfies the condition for constructive interference, the **Circular Fringe** is bright; otherwise it is dark.
- **Localised Fringes:** When the system is illuminated with a parallel light beam, the reflected rays are not parallel. They interfere near to the top surface of the film. When viewed from the top, the rays appear to diverge. As the fringes are seen at the upper surface of the film, they are said to be localised in the film.
- **White Light:** With white light, few **Coloured Fringes** are seen at centre. Away from centre they overlap.

2.7 APPLICATIONS OF NEWTON'S RINGS

2.7.1 Determination of Wavelength of Incident Light or Radius of Curvature of Plano-Convex Lens

- The experimental arrangement is shown in Fig. 2.9 (b). Let R be the radius of curvature of the lens and λ the wavelength of the light used. If D_n is the diameter of the nth dark rings, then

$$D_n^2 = 4 n \lambda R \qquad \text{... (2.42)}$$

Similarly, for (n + p)th dark rings,

$$D_{n+p}^2 = 4 (n + p) \lambda R \qquad \text{... (2.43)}$$

Subtracting (2.42) from (2.43), we get,

$$D_{n+p}^2 - D_n^2 = 4 p \lambda R$$

$$\therefore \quad \lambda = \frac{D_{n+p}^2 - D_n^2}{4 p R} \qquad \text{... (2.44)}$$

- The microscope is adjusted to obtain Newton's rings. The centre of the cross wire is made to coincide with the central dark fringe. Counting the central fringe as n = 0, the cross wire is moved to nth and (n + p)th dark fringe to the left and position of microscope is noted on micrometer screw gauge.
- In the same way position of nth and (n + p)th fringe is noted on right. Subtracting position on left and right for nth and (n + p)th fringe gives diameter of nth and (n + p)th fringe respectively.
- Radius of curvature '**R**' is found using a spherometer.
- The wavelength λ of monochromatic source of light is found using relation (2.44). If λ is known, then same relation may be used to find R.

2.7.2 Determination of Refractive Index of a Liquid

- Firstly, perform the experiment when there is an air film between the glass plate and plano-convex lens. The system is placed in a metal container. The diameter of nth and (n + p)th dark rings are determined using a travelling microscope.

For air,

$$D_{n+p}^2 - D_n^2 = 4 p \lambda R \qquad \text{... (2.45)}$$

- Pour the liquid, whose refractive index is to be determined, in the container without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. Now, measure the diameter of the nth and (n + p)th dark rings.

- For liquid, we have $2\mu t \cos (r + \alpha) = n\lambda$ as the condition for darkness.

  For normal incidence, $r = 0$ and $\alpha = 0$.

$$\therefore \quad 2\mu t = n\lambda \qquad \text{... (2.46)}$$

$$\text{But} \quad t = \frac{r'^{2}_{n}}{2R} = \frac{D'^{2}_{n}}{8R} \qquad \text{... (2.47)}$$

From (2.46) and (2.47),

$$2\mu \cdot \frac{D^{2}_{n}}{8R} = n\lambda$$

$$D^{2}_{n} = \frac{4n\,\lambda R}{\mu} \qquad \text{... (2.48)}$$

- If D'^{2}_{n} and D'^{2}_{n+p} are the diameters of nth and (n + p)th dark rings in liquid, then

$$D'^{2}_{n} = \frac{4\,n\,\lambda R}{\mu} \qquad \text{... (2.49)}$$

$$D'^{2}_{n+p} = \frac{4\,(n+p)\,\lambda R}{\mu} \qquad \text{... (2.50)}$$

Subtracting (2.49) from (2.50), we get

$$D'^{2}_{n+p} - D'^{2}_{n} = \frac{4\,p\,\lambda R}{\mu} \qquad \text{... (2.51)}$$

From equations (2.45) and (2.51),

$$\mu = \frac{D^{2}_{n+p} - D^{2}_{n}}{D'^{2}_{n+p} - D'^{2}_{n}} \qquad \text{... (2.52)}$$

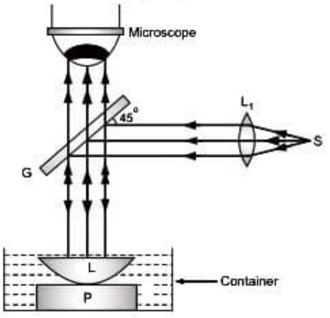


Fig. 2.12: Experimental arrangement for measurement of R.I. of liquid

Problem 2.23: *A convex lens is placed on a plane glass slab and is illuminated by a monochromatic light. The diameter of the 10th dark ring is measured and is found to be 0.433 cm. The radius of curvature of the lower surface of the lens is 70 cm. Find the wavelength of the light used.*

Data: $R = 70$ cm, $n = 10$, $D_n = 0.433$ cm

Formula: $D^{2}_{n} = 4\,n\,R\,\lambda$

Solution: $(0.433)^2 = 4 \times 10 \times 70 \times \lambda$

$$\therefore \quad \lambda = \frac{(0.433)^2}{4 \times 10 \times 70}$$

$$\lambda = 6.696 \times 10^{-5}\ \text{cm}$$

$$= \boxed{6696\ \text{A}^\circ}$$

Problem 2.24: *In a Newton's rings experiment, the diameter of the 15th dark ring was found to be 0.590 cm and that of the 5th dark ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of the light used.*

Data: $D_{15} = 0.590$ cm, $D_5 = 0.336$ cm, $R = 100$ cm, $m = 10$

Formula: $$\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR}$$

Solution: $$\lambda = \frac{D^{2}_{15} - D^{2}_{5}}{4 \times 10 \times R} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$\lambda = 5.880 \times 10^{-5}\ \text{cm}$$

$$\boxed{\lambda = 5880\ \text{A}^\circ}$$

Problem 2.25: *The diameter of a dark ring in Newton's rings experiment decreases from 1.4 cm to 1.2 cm when air is replaced by a liquid as medium between lens and flat surface. Calculate the refractive index of the liquid.*

Data: $D_{air} = 1.4$ cm, $D_{liquid} = 1.2$ cm

Formula: $$\mu = \frac{D^{2}_{air}}{D^{2}_{liquid}}$$

Solution: $$\mu = \frac{(1.4)^2}{(1.2)^2}$$

$$\boxed{\mu = 1.36}$$

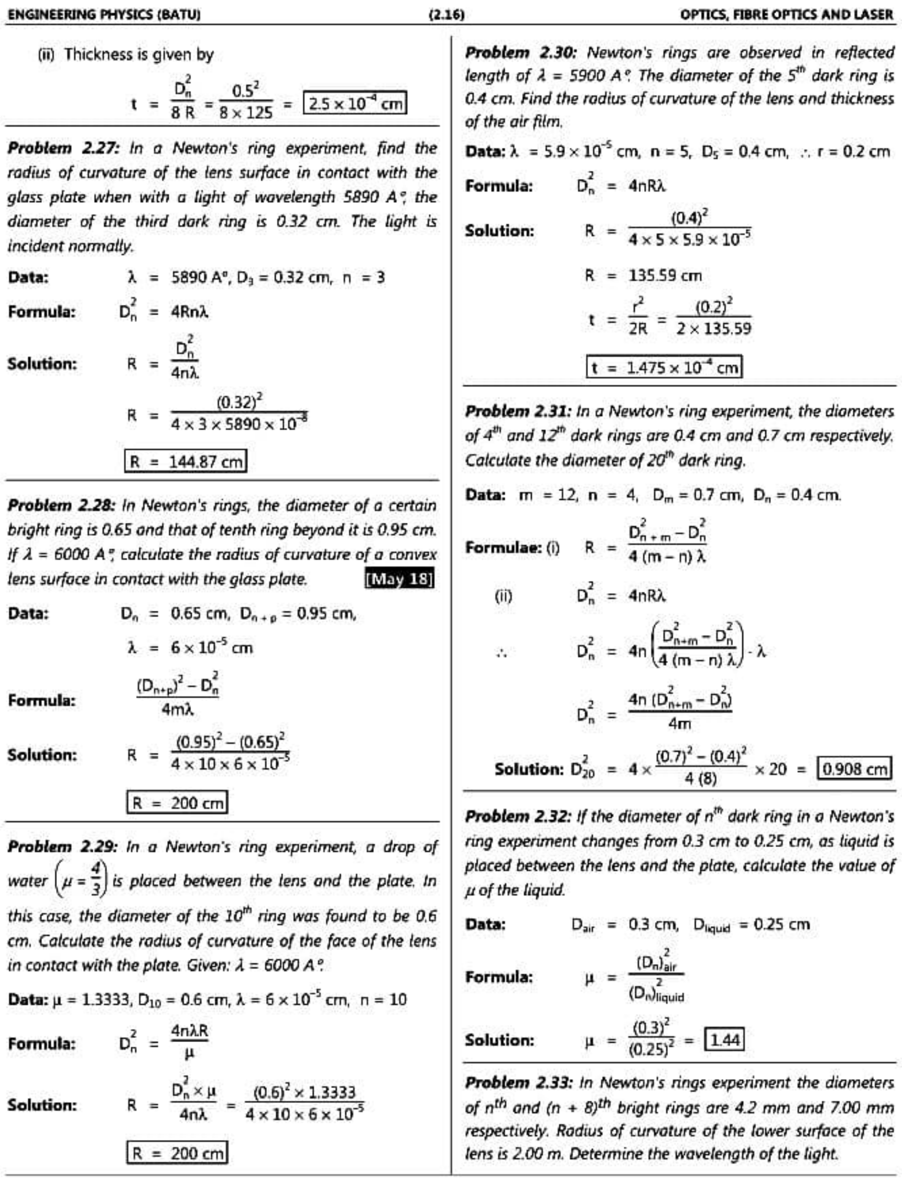
Problem 2.26: *The diameter of the tenth dark ring in Newton's rings experiment is 0.5 cm. Calculate the radius of curvature of the lens and the air thickness at the position of the ring. The wavelength of light used is 5000 A°.*

Data: $D_{10} = 0.5$ cm, $n = 10$, $\lambda = 5000 \times 10^{-8}$ cm

Formulae: (i) $D^{2}_{n} = 4nR\lambda$, (ii) $t = \dfrac{D^{2}_{n}}{8\,R}$

Solution: (i) $$R = \frac{D^{2}_{n}}{4\,n\,\lambda} = \frac{0.5^2}{4 \times 10 \times 5000 \times 10^{-8}}$$

$$\boxed{R = 125\ \text{cm}}$$



(ii) Thickness is given by

$$t = \frac{D_n^2}{8\,R} = \frac{0.5^2}{8 \times 125} = \boxed{2.5 \times 10^{-4}\ \text{cm}}$$

Problem 2.27: *In a Newton's ring experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890 A°, the diameter of the third dark ring is 0.32 cm. The light is incident normally.*

Data: $\lambda = 5890\ \text{A}^\circ,\ D_3 = 0.32\ \text{cm},\ n = 3$

Formula: $D_n^2 = 4Rn\lambda$

Solution:

$$R = \frac{D_n^2}{4n\lambda}$$

$$R = \frac{(0.32)^2}{4 \times 3 \times 5890 \times 10^{-8}}$$

$$\boxed{R = 144.87\ \text{cm}}$$

Problem 2.28: *In Newton's rings, the diameter of a certain bright ring is 0.65 and that of tenth ring beyond it is 0.95 cm. If λ = 6000 A°, calculate the radius of curvature of a convex lens surface in contact with the glass plate.* **[May 18]**

Data:

$$D_n = 0.65\ \text{cm},\ D_{n+p} = 0.95\ \text{cm},$$

$$\lambda = 6 \times 10^{-5}\ \text{cm}$$

Formula:

$$\frac{(D_{n+p})^2 - D_n^2}{4m\lambda}$$

Solution:

$$R = \frac{(0.95)^2 - (0.65)^2}{4 \times 10 \times 6 \times 10^{-5}}$$

$$\boxed{R = 200\ \text{cm}}$$

Problem 2.29: *In a Newton's ring experiment, a drop of water $\left(\mu = \frac{4}{3}\right)$ is placed between the lens and the plate. In this case, the diameter of the 10th ring was found to be 0.6 cm. Calculate the radius of curvature of the face of the lens in contact with the plate. Given: λ = 6000 A°.*

Data: $\mu = 1.3333,\ D_{10} = 0.6\ \text{cm},\ \lambda = 6 \times 10^{-5}\ \text{cm},\ n = 10$

Formula:

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

Solution:

$$R = \frac{D_n^2 \times \mu}{4n\lambda} = \frac{(0.6)^2 \times 1.3333}{4 \times 10 \times 6 \times 10^{-5}}$$

$$\boxed{R = 200\ \text{cm}}$$

Problem 2.30: *Newton's rings are observed in reflected length of λ = 5900 A°. The diameter of the 5th dark ring is 0.4 cm. Find the radius of curvature of the lens and thickness of the air film.*

Data: $\lambda = 5.9 \times 10^{-5}\ \text{cm},\ n = 5,\ D_5 = 0.4\ \text{cm},\ \therefore r = 0.2\ \text{cm}$

Formula: $D_n^2 = 4nR\lambda$

Solution:

$$R = \frac{(0.4)^2}{4 \times 5 \times 5.9 \times 10^{-5}}$$

$$R = 135.59\ \text{cm}$$

$$t = \frac{r^2}{2R} = \frac{(0.2)^2}{2 \times 135.59}$$

$$\boxed{t = 1.475 \times 10^{-4}\ \text{cm}}$$

Problem 2.31: *In a Newton's ring experiment, the diameters of 4th and 12th dark rings are 0.4 cm and 0.7 cm respectively. Calculate the diameter of 20th dark ring.*

Data: $m = 12,\ n = 4,\ D_m = 0.7\ \text{cm},\ D_n = 0.4\ \text{cm}.$

Formulae: (i)

$$R = \frac{D_{n+m}^2 - D_n^2}{4\,(m-n)\,\lambda}$$

(ii)

$$D_n^2 = 4nR\lambda$$

$$\therefore \quad D_n^2 = 4n\left(\frac{D_{n+m}^2 - D_n^2}{4\,(m-n)\,\lambda}\right) \cdot \lambda$$

$$D_n^2 = \frac{4n\,(D_{n+m}^2 - D_n^2)}{4m}$$

Solution:

$$D_{20}^2 = 4 \times \frac{(0.7)^2 - (0.4)^2}{4\,(8)} \times 20 = \boxed{0.908\ \text{cm}}$$

Problem 2.32: *If the diameter of nth dark ring in a Newton's ring experiment changes from 0.3 cm to 0.25 cm, as liquid is placed between the lens and the plate, calculate the value of μ of the liquid.*

Data: $D_{air} = 0.3\ \text{cm},\ D_{liquid} = 0.25\ \text{cm}$

Formula:

$$\mu = \frac{(D_n)_{air}^2}{(D_n)_{liquid}^2}$$

Solution:

$$\mu = \frac{(0.3)^2}{(0.25)^2} = \boxed{1.44}$$

Problem 2.33: *In Newton's rings experiment the diameters of nth and (n + 8)th bright rings are 4.2 mm and 7.00 mm respectively. Radius of curvature of the lower surface of the lens is 2.00 m. Determine the wavelength of the light.*

Data:

$D_n = 4.2$ mm

$D_{n+8} = 7$ mm

$R = 2$ m

Formula:

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4\,m\,R}$$

Solution:

$$\lambda = \frac{D_{n+8}^2 - D_n^2}{4\ (n+8-n)\,R}$$

$$\lambda = \frac{0.7^2 - 0.42^2}{4 \times 8 \times 200}$$

$$\lambda = 5 \times 10^{-5}\ \text{cm}$$

$$\boxed{\lambda = 5000\ \text{A}^\circ}$$

Problem 2.34: *Newton's rings are formed by light reflected normally from a plano-convex lens and a plane glass plate with a liquid between them. The diameter of n^{th} ring is 2.18 mm and that of $(n + 10)^{th}$ ring is 4.51 mm. Calculate the refractive index of the liquid, given that the radius of curvature of the lens is 90 cm and wavelength of light is 5893 A°.*

Data:

$D'_n = 2.18$ mm

$D'_{n+10} = 4.5$ mm

$R = 90$ cm

$\lambda = 5893$ A°

Formula:

$$R = \frac{\mu\,(D'^2_{n+m} - D'^2_n)}{4m\lambda}$$

$$\mu = \frac{4\,m\,\lambda\,R}{D'^2_{n+m} - D'^2_n}$$

Solution:

$$\mu = \frac{4 \times 10 \times 5893 \times 10^{-8} \times 90}{0.45^2 - 0.218^2}$$

$$\boxed{\mu = 1.368}$$

Problem 2.35 : *In a Newton's rings experiment, the diameter of the 5^{th} ring was 0.336 cm and that of 15^{th} ring was 0.59 cm. Find the radius of curvature of the plano-convex lens, if the wavelength of light used is 5890 A°.*

Data:

$D_{15} = 0.59$ cm

$D_5 = 0.336$ cm

$\lambda = 5890$ A°

$m = 10$

Formula:

$$R = \frac{D_{n+m}^2 - D_n^2}{4\,m\,\lambda}$$

Solution:

$$R = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times \lambda}$$

$$R = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

$$\boxed{R = 99.83\ \text{cm}}$$

2.8 INTRODUCTION TO POLARIZATION

- The phenomenon like interference or diffraction prove the wave nature of light. But it does not tell us whether the light waves are longitudinal or transverse. Because even longitudinal waves, like sound waves, show the phenomena of interference and diffraction.
- The important difference between longitudinal and transverse wave is that the transverse waves can be polarized.
- The phenomenon of polarization can be explained only by considering the transverse nature of light. And it has been proved by electromagnetic theory that the light is transverse wave.

2.9 POLARIZATION OF WAVES

- The transverse nature of waves leads to the characteristic phenomenon called **Polarization**. The characteristic, polarization is not exhibited by longitudinal waves. Thus only transverse waves could be polarized.
- In a transverse wave, if the directions of all the vibrations at all the points are restricted to one particular plane, then the wave is called **Polarized**, more specific plane polarized. A plane polarized wave is the simplest of a transverse wave, which is also termed as **Linearly Polarized Wave**.

2.10 REPRESENTATION OF POLARIZED LIGHT

[May 18]

- According to the electromagnetic theory, light consists of electric and magnetic vectors vibrating continuously with time in a plane, transverse to the direction of propagation of light and to each other. However, **in explaining polarization only the vibrations of the electric vector are considered.**
- It does not mean that magnetic field vectors are absent, they are present. But for drawing simplicity they are not shown in the diagram.

(i) Unpolarized Light

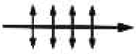
- The light having vibrations along all possible directions perpendicular to the direction of propagation of light, is called an **Unpolarized Light**. The vibrations are symmetrical about the direction of propagation of light.

Fig. 2.13: Unpolarized light

- It can be considered to consist an infinite number of waves each having its own vibration. Since unpolarized light has vibrations along all possible directions, at right angles to the directions of propagation of light, it is represented by a star.

(ii) Polarized Light

- The light having vibration only along a single plane perpendicular to the direction of propagation of light is called a **Polarized Light**. It's vibrations are one sided, therefore it is dissymmetrical about the direction of propagation of light.
- The polarized beam of light has vibrations along a single plane. If they are parallel to the plane of the paper, they are represented by arrows [See Fig. 2.14 (a)]. If they are perpendicular to the plane of the paper, they are represented by dots on a ray of light. [See Fig. 2.14 (b)].



(a) Parallel to plane of paper

(b) Perpendicular to plane of paper

Fig. 2.14: Plane polarized light

(iii) Partially Polarized Light

- A partially polarized light is a mixture of plane polarized and unpolarized light. It is represented as shown in Fig. 2.15.
- In partially polarized light the vibrations in the plane of plane polarized light dominate over the vibrations in other directions.

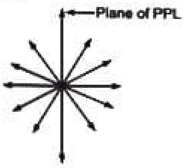


Fig. 2.15: Partially polarized light

2.11 METHODS OF PRODUCTION OF POLARIZED LIGHT

- Although polarized light has many applications in science and engineering, but the light available naturally is unpolarized. So different methods have been developed to obtain polarized light artificially.
- Every method uses one or the other optical phenomena like reflection, refraction, scattering, double refraction etc., for getting polarized light.
- Here we will be learning some of the methods for obtaining plane polarized light.

2.11.1 Production of Plane Polarized Light by Reflection [May 18]

- Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass.

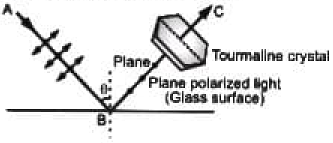


Fig. 2.16

- Consider the light incident along the path AB on the glass surface. A part of light is reflected along BC. In the path of BC, place a tourmaline crystal and rotate it slowly. It is observed that light is completely extinguished only at one particular angle of incidence.
- At any other angle of incidence there is preferential reflection of the components having vibrations perpendicular to the plane of incidence.
- This angle of incidence is equal to 57.5° for a glass surface and is known as the **Polarization Angle**.
- The vibrations of the incident light can be resolved into components-parallel to the reflecting surface (glass surface) and perpendicular to the reflecting surface. Light due to the components parallel to the reflecting surface is reflected whereas light due to the components perpendicular to the reflecting surface is transmitted i.e. the plane of the vibrations of reflecting rays are at right angles to the plane of incidence and the plane of vibrations of refracted rays are in the plane of incidence. Thus, light reflected by the surface is polarised in the plane of incidence and can be detected by tourmaline crystal.

Note

- If light is polarised perpendicular to the plane of incidence, it means that vibrations are in the plane of incidence.
- If light is polarised in the plane of incidence, it means that vibrations are perpendicular to the plane of incidence.

Polarizing Angle or Angle of Polarization

- It is defined as that angle of incidence on the reflecting surface for which reflected light is completely plane polarized.
- As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelength. Therefore, polarising angle will be complete only for light of a particular wavelength at a time i.e. for monochromatic light (for a given surface).

Brewster's Law

In 1811, Sir David Brewster found that ordinary light is completely polarised in the plane of incidence when it gets itself reflected from a transparent medium at a particular angle known as the **Polarizing Angle**.

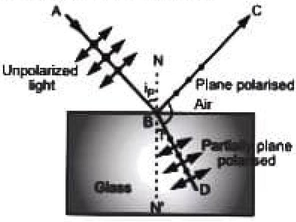


Fig. 2.17

- He was able to prove that, the tangent of the angle of polarisation is numerically equal to the refractive index of the medium. i.e. $\mu = \tan i_p$.
- Consider unpolarised light is incident on the glass surface at the polarising angle. It is reflected along BC and refracted along BD.

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \qquad \text{... (2.53)}$$

From Brewster's law,

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p} \qquad \text{... (2.54)}$$

Comparing equations (2.53) and (2.54),

$$\cos i_p = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \quad i_p = \frac{\pi}{2} - r$$

$$i_p + r = \frac{\pi}{2}$$

$$\text{As} \quad i_p + r = \frac{\pi}{2},$$

$$\angle CBD = \frac{\pi}{2}$$

Therefore, reflected and refracted rays are at right angles to each other.

2.11.2 Production of Plane Polarized Light by Refraction: Pile of Plates

- When unpolarized light is incident at an polarizing angle on a transparent surface the reflected light is polarized completely whereas refracted light is partially polarized.

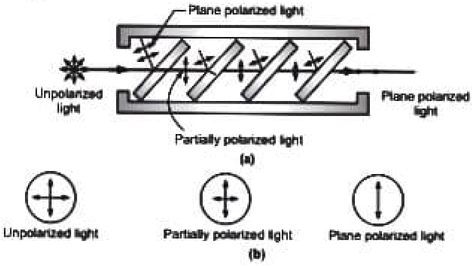


Fig. 2.18: Polarization by refraction

- If more than one refracting surface i.e. stack of glass plates are used in place of one, the process is repeated. At every surface, the unpolarized component decreases, thus the polarized component becomes prominent, giving almost plane polarized light in the direction parallel to the pile of plates.
- A pile of plates contains about 15 glass plates placed in a metal tube of suitable size. The plates are kept at 33° with the axis of tube so that the incident unpolarized light is incident at polarizing angle at the first plate.
- The unpolarized light entering in the tube will undergo successive reflection and refraction such that the emerging ray is plane polarized light.

2.11.3 Double Refraction [Dec. 17]

- The phenomenon of double refraction was discovered by Erasmus Bartholinus in 1669 during his studies on calcite. When light is incident on a calcite crystal, it is found to produce two refracted rays which are different in properties. The phenomenon of causing **Two Refracted Rays** by a crystal is called **Birefringence** or **Double Refraction**. The crystals are said to be **Birefringent**.

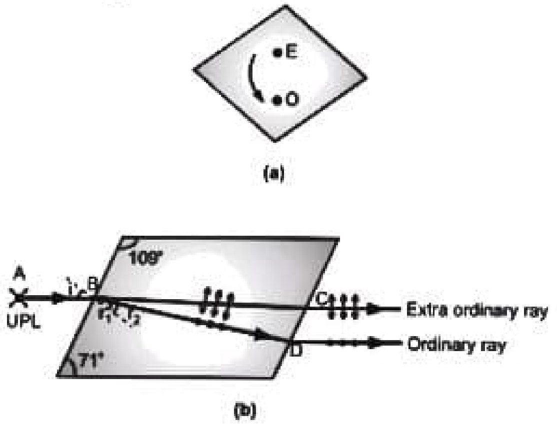


Fig. 2.19: Double reflection

- All anisotropic materials exhibit double refraction. The two rays formed in double refraction are linearly polarized in mutually perpendicular directions.
- One of the rays **obeys Snell's Law** of refraction and hence is called **an Ordinary Ray or O-ray**. The other ray **does not obey Snell's Law** and is called an **Extraordinary Ray or E-Ray.** Both of them are linearly/plane polarised, but plane of polarisation is perpendicular to each other. If one of the rays is eliminated, the light transmitted by the crystal will be a linearly/plane polarized light.
- When a ray of light AB is incident on the calcite crystal making an angle of incidence i, it is refracted along two paths inside the crystal: (i) along BC making an angle of refraction r_2, (ii) along BD making an angle of refraction r_1. These two rays emerge out along DO and CE which are parallel as the crystal faces are parallel.

Optic Axis

- The **Optic Axis** is the direction of symmetry of unisotropic media along which double refraction does not take place.
- A line drawn through any of the blunt corners making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore, optic axis is not a line but **It Is a Direction.**

Principal Section

- A plane containing **The Optic Axis** and **Perpendicular to the Opposite Faces** of the crystal is called the **Principal Section Of The Crystal.** The principal section cuts the surfaces of a calcite crystal in a parallelo-gram with angles 109° and 71°.

Fig. 2.20: Principal section of calcite crystal

Principal Plane

- The plane containing the optic axis and the ordinary ray is called principal plane of the ordinary ray. Similarly the plane containing the optic axis and the extraordinary ray is called the principal plane of the extraordinary ray.
- Experiments revealed that the vibrations of the ordinary rays are perpendicular to the principal section of the crystal while the vibrations of the extraordinary rays are parallel to the principal section of the crystal. Thus, the two rays are plane polarized, their vibrations being at right angles to each other.

2.11.4 Polarization by Double Refraction - Nicol Prism

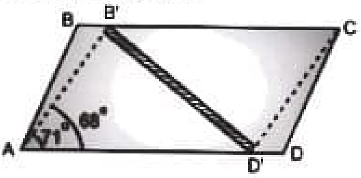
- Nicol prism is an optical device used for producing and analysing plane polarised light.

Principle

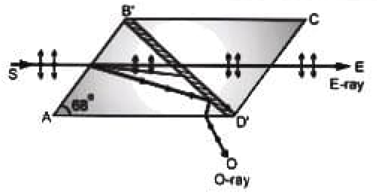
- The Nicol prism is made in such a way that it eliminates one of the refracted rays by total internal reflection i.e. O-ray is eliminated and only E-ray is transmitted through the prism.

Construction

A calcite crystal whose length is three times it's breadth is taken. Let ABCD be the principal section of the crystal with ∠ BAD = 71°. The end faces of the crystal are cut in such a way that they make angles 68° and 112° in the principal section instead of 71° and 109°.



(a)



(b)

Fig. 2.21

- The crystal is then cut into two pieces from one blunt corner to the other along a plane perpendicular to the principal section. The two cut faces are grounded and polished optically flat. It is then cemented together by Canada balsam whose refractive index lies between the refractive indices for the O-ray and E-ray for calcite.

  Refractive index of Calcite for O-ray

  $$\mu_o = 1.658$$

  Refractive index of Canada balsam

  $$\mu_c = 1.55 \text{ Using sodium light of } \lambda = 5893 \text{ A}^\circ,$$

  Refractive index of Calcite for E-ray

  $$\mu_e = 1.486$$

- Canada balsam layer acts as a rarer medium for O-ray and as a denser medium for E-ray. Except the end faces, the sides of the crystal are blackened.

Working

- When a ray of unpolarized light is incident on the prism surface, it splits into O-ray and E-ray. Both the rays are polarized having vibrations at right angles to each other.
- When the O-ray passes from a portion of the crystal into the layer of Canada balsam, it passes from a denser medium to rarer medium. When the angle of incidence is greater than the critical angle, the O-ray is totally internally reflected and is not transmitted.
- When the E-ray passes from calcite to the Canada balsam layer, it enters in rarer medium. Therefore, the E-ray is not affected and is transmitted through the prism.

  Refractive index for O-ray with respect to Canada balsam,

  $$\mu = \frac{1.658}{1.55}$$

  If C is the critical angle,

  $$\therefore \quad \mu = \frac{1}{\sin C}$$

  $$\sin C = \frac{1}{\mu} = \frac{1.55}{1.658}$$

  $$C = 69^\circ$$

- As the length of the crystal is large, the angle of incidence at Canada balsam surface for the O-ray is greater than the critical angle. Thus, it suffers total internal reflection while E-ray is transmitted which is plane polarized having vibrations in the principal section.

Special Cases

- If the angle of incidence is less than the critical angle for O-ray, it is not reflected and is transmitted through the prism. In this position, both the O-ray and E-ray are transmitted through the prism.
- The E-ray also has a limit beyond which it is totally internally reflected by Canada balsam surface. If E-ray travels along the optic axis, its refractive index is the same as that of O-ray i.e. 1.658. But it is 1.486 for all other directions of E-ray. Therefore depending on the direction of propagation of E-ray, μ_e lies between 1.486 and 1.658. Therefore for a particular case, μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then E-ray will also be totally internally reflected.

2.12 HUYGEN'S THEORY OF DOUBLE REFRACTION [Dec. 18]

Huygen explained the phenomenon of double refraction on the basis of the principle of secondary wavelets.

He assumed:

- When a beam of ordinary unpolarized light strikes a doubly refracting crystal, each point on the surface sends out **Two Wavefronts**, one for ordinary ray and the other for extraordinary ray.
- The **Ordinary-Ray** travels with the **Same Speed** v_o in all directions and the crystal has a single refractive index $\mu_o = \frac{c}{v_o}$ for this wave. Thus, the O-ray has a **Spherical Wavefront**.
- The **Speed of Extra-Ordinary Ray V_e Varies with Direction**. So, the refractive index, $\mu_e = \frac{c}{v_e}$ also varies with direction for the E-ray. Therefore, the extra-ordinary ray develops a wavefront which is **Ellipsoidal**.

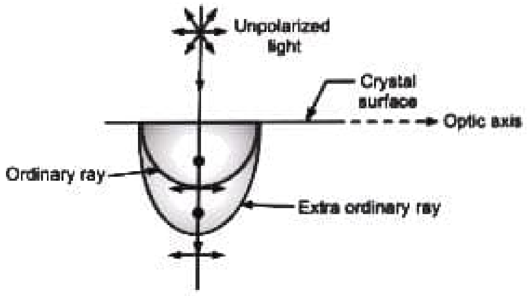
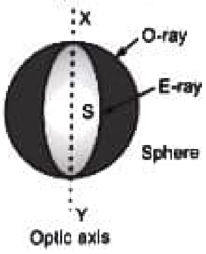
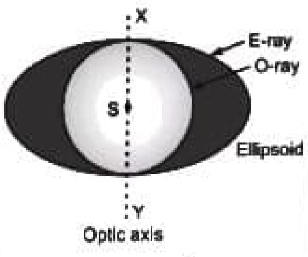


Fig. 2.22: Double refraction

- The velocity v_e measured is perpendicular to the optic axis.
- The velocities of the O-ray and E-ray are the same along the optic axis.
- When rays are incident along the optic axis, the spherical and ellipsoidal wavefronts touch each other at points of intersection with the optic axis and double refraction does not take place.
- If $v_o > v_e$ or $\mu_o < \mu_e$, the spherical wavefront lies outside the elliptical wavefront. Such crystals are called **Positive Crystals**. The examples of positive crystal are quartz, ice etc.
- If $v_e > v_o$ or $\mu_e < \mu_o$, the elliptical wavefront lies outside the spherical wavefront. Such crystals are called **Negative Crystals**. The examples of negative crystals are calcite, tourmaline, etc.

2.12.1 Positive and Negative Crystals

Positive Crystals	Negative Crystals
1. For positive crystals, $v_o > v_e$ and $\mu_o < \mu_e$.	1. For negative crystals, $v_o < v_e$ and $\mu_o > \mu_e$.
2. The velocity of O-ray is same in all directions.	2. The velocity of O-ray is same in all directions.
3. The wavefront of O-ray lies outside the wavefront of E-ray.	3. The wavefront of O-ray lies inside the wavefront of E-ray.
4. Examples: Quartz, Ice.  Fig. 2.23 (a)	4. Examples: Calcite, Tourmaline.  Fig. 2.23 (b)

2.13 CASES OF DOUBLE REFRACTION OF CRYSTAL CUT WITH OPTIC AXIS LYING IN THE PLANE OF INCIDENCE

2.13.1 Parallel to the Surface

- Fig. 2.24 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. The optic axis lies along XY and is in the plane of incidence.
- At the points A and B, it develops two wavefronts, one spherical for O-ray and one elliptical for E-ray. The envelope of O-ray and E-ray gives the corresponding wavefront which is plane polarized.
- It should be noted that both O-ray and E-ray are plane polarized light. Here both O-ray and E-ray travel along the same direction with different velocities. As O-ray and E-ray travel along the same direction with different velocities, a path difference is introduced between them.
- This principle is used in the construction of quarter and half-wave plates.

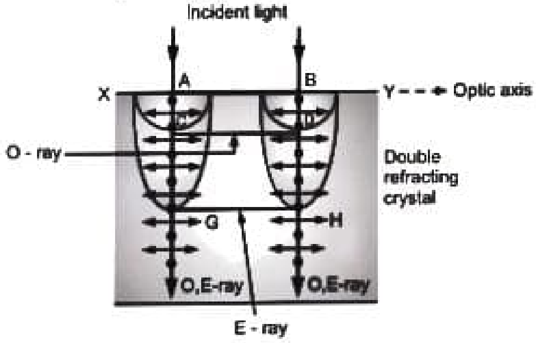


Fig. 2.24

2.13.2 Perpendicular to the Surface

- Fig. 2.25 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. Optic axis lies in the plane of incidence and perpendicular to the crystal surface.
- As the light is incident in the direction of optic axis, O-ray and E-ray travel with the same speed along the optic axis. As a result O-ray and E-ray travel along the same directions with same velocity. Hence the phenomenon of **Double Refraction is Absent** in this case. Ordinary and extraordinary wavefronts CD and GH coincide at all instants.

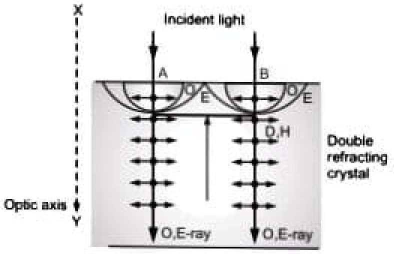


Fig. 2.25

2.13.3 Inclined to the Surface

- Fig. 2.26 shows an unpolarized plane wavefront incident normally on the crystal surface so that the optic axis makes an angle with the crystal surface.
- O-ray and E-ray travel with different velocities in different direction in the crystal. Hence double refraction is seen in this case and both O-ray and E-ray are separated by an angle depending upon the distance travelled in crystal.

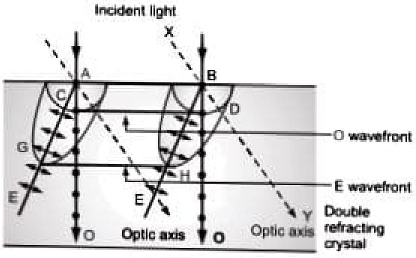


Fig. 2.26

2.14 LAURENT'S HALF SHADE POLARIMETER

2.14.1 Optical Activity

- When a beam of a plane polarized light is directed along the optic axis of quartz, the plane of polarization turns steadily about the direction of the beam and the beam emerges vibrating in some other plane than that at which it has entered.
- The amount of rotation depends upon the distance travelled in the medium and wavelength of the light. This phenomenon of rotation of the plane of polarization is called **Optical Activity**. The substances which show optical activity are sodium chlorate, turpentine, sugar crystal etc. Fig. 2.27 shows optical activity.

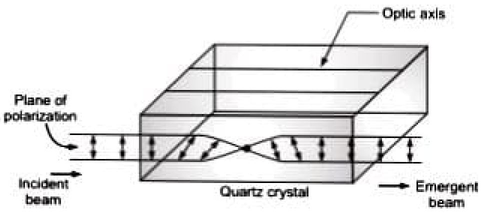


Fig. 2.27: Optical activity

- Some crystals rotate the plane of vibration to the right and some to the left. The substances which rotate to the right are called **Right Handed or Dextro-Rotatory** and those which rotate to the left are called **Left Handed** or **Laevo-Rotatory**.

2.14.2 Specific Rotation

- A striking feature of optical activity is that different colours are rotated by different amount. This rotation is nearly proportional to the inverse square of the wavelength. This gives a **Rotatory Dispersion**, violet being rotated nearly four times as much as red light. Fig. 2.28 shows rotatory dispersion.

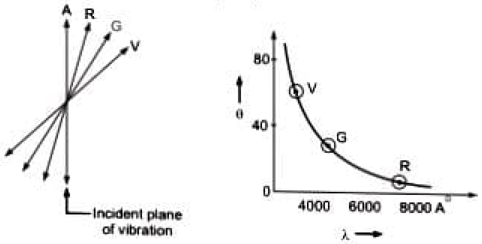


Fig. 2.28: Rotatory dispersion

- The rotation for a 1 mm thick plate is called the **Specific Rotation**.

2.14.3 Optically Active Materials

- Optical activity is exhibited by organic compounds whose molecular arrangement lacks in symmetry. Therefore, upon entering in the material, the plane polarized light changes the plane of polarization depending upon the molecular arrangement of the material.
- Most of the petroleum exhibits optical activity which are organic in nature. The optical activity is not exhibited by synthetic materials as they are mixture of left handed and right handed molecules in equal quantity, thus giving net zero rotation.

2.14.4 Laurent's Half Shade Polarimeter

- Polarimeters are instruments, used for finding the optical rotation of different solutions. When they are calibrated to read directly the percentage of cane sugar in a solution, they are named as saccharimeters.
- Polarimeters can be used to find the specific rotation of sugar solution or if the specific rotation is known, they can be used to find its concentration.

Construction :

- The essential parts of a polarimeter are as shown in Fig. 2.29. Light from a monochromatioc source S is rendered parallel by a collimating lens L. N_1 and N_2 are two Nicol prisms, N_1 acts as a polarizer while N_2 acts as a analyzer. N_2 is capable of rotation about a common axis of N_1 and N_2. The rotation of N_2 can be read on a graduated circular scale S.C. The light after passing through the polarizer N_1 becomes plane polarized with its vibrations in the principal plane of the Nicol N_1.
- The plane polarized light now passes through a half shade device HS and then through a glass tube BC containing the optically active substance. The tube is closed at the end by metal covers. The light emergent from the analyzer N_2 is viewed through a telescope T. The telescope is focused on the half shade.

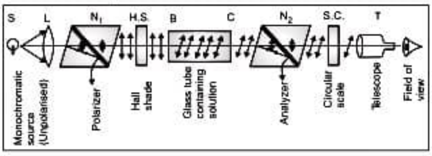


Fig. 2.29 : Laurent's half shade polarimeter

Action of Half Shade

- When an optically active substance is placed in between two crossed Nicols, the field of view is not dark. In order to make it dark the analyzer is rotated. It is observed that, when the analyzer is rotated, the field of view is not dark for a considerable region. Hence the measurement of optical is not accurate. To avoid this difficulty, a half-shade device is used. Laurent's half-shade plate consists of a semi-circular half wave plate ACB of quartz.
- The thickness of the quartz is so chosen that it introduces a phase difference of π between the ordinary and extraordinary ray passing through it. The other half ADB is made of glass and its thickness is such that is absorbs and transmits the same amount of light as done by the quartz half-plate. The two plates are cemented along the diameter AB. The optic axis of the quarts plate lies along the line AB.

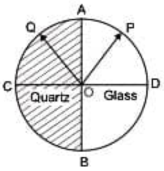


Fig. 2.30 : Laurent's half shade plate

- Let the plane polarized light coming from the polarizer be incident normally on the half shade the plate with its vibrations parallel to OP. Here OP makes an angle θ with AB. The vibrations emerge from glass plate along the plane OP. Inside the quartz plate, the incident ray would be split up into two ordinary and extraordinary components. One having vibrations along OA and the other along OD. These rays travel with unequal velocities through the quartz plate which introduces a phase difference of π between them.
- Hence on emerging from the plate, the vibrations will be along OA and PC and their resultant vibrations along OQ, where ∠ AOOP = ∠ AOQ. If the initial position of ordinary components is represented by OD then the final position is represented by OC. If the principal plane of the analyzing Nicol is parallel to OP, then the light emerging from glass portion will pass unobstructed while light from quartz will be partly obstructed.
- Due to this fact, the glass half will appear brighter then the quartz half. On the other hand, if the principal plane of the analyzer is parallel to OQ, the light from quartz portion will be unobstructed while light from glass will be partly obstructed.
- Thus, the quartz half will appear brighter than the glass half. The two halves will look equally illuminated when the analyzer is so turned that its principal plane is exactly parallel to AB. Any slight rotation in either direction produces a sharp difference in the illumination of the two halves.

Determination of Specific Rotation

- Specific rotation S is given by

$$S = \frac{\theta}{l \times c}$$

where θ is the angle of rotation in degrees, l, is the length of the solution in decimeters and c is the concentration of solution in gm/cc. Hence, to determine the specific rotation of a substance, a solution of known concentration is prepared. The length of the solution is measured directly. The value of θ is determined as follows :

- The experimental tube is filled with distilled water and placed in its position. The telescope is focused on the half-shade plane and the analyzer is rotated till equally bright position is observed in the field of view.
- The readings of two verniers on the circular scale is noted. Now, the tube is filled with the optically active solution and placed in its position. The analyzer is rotated and is brought to a position so that the whole field of view is equally bright. The new positions of the two verniers are again noted on the circular scale.
- The difference in the two readings of the same vernier gives the angle of rotation, θ produced by the solution. Thus knowing θ, l and c the specific rotation S can be calculated by the given formula.

Problem 2.36: *Two polarizing plates have polarizing directions parallel so as to transmit maximum intensity of light. Through what angle must either plate be turned if the intensity of the transmitted beam is to drop to one third ?*

Data: $I = \frac{I_0}{3}$

Formula: From Law of Malus, $I = I_0 \cos^2 \theta$

Solution:

Substituting, $\frac{I_0}{3} = I_0 \cos^2 \theta$

$$\cos^2 \theta = \frac{1}{3}$$

Or $\cos \theta = \pm \frac{1}{\sqrt{3}}$

$$\boxed{\theta = 54° \, 41' \text{ or } \pm 144° \, 40'}$$

Problem 2.37: *At a certain temperature, the critical angle of incidence of water for total internal reflection is 48° for a certain wavelength. What is the polarizing angle and the angle of refraction for light incident on the water that gives maximum polarization of the reflected light ?*

Data: Critical angle $C = 48°$

Formulae: (i) $\mu = \frac{1}{\sin C}$, (ii) $\mu = \tan i_p$

Solution:

(i) Substituting, $\mu = \frac{1}{\sin 48°}$

$$\boxed{\mu = 1.345}$$

(ii) From Brewster's law,

$$\mu = \tan i_p$$

$$1.345 = \tan i_p$$

$$i_p = \tan^{-1} (1.345)$$

$$i_p = 53° \, 22'$$

But $i_p + r = 90°$

∴ $r = 90° - i_p$

$90° - 53° \, 22' = r$

∴ $\boxed{r = 36° \, 38}$

Problem 2.38: *Two Nicol prisms are oriented with their principal planes making an angle of 60°. What percentage of incident unpolarized light will pass through the system ?*

Data: $\theta = 60°$

Formulae: (i) For unpolarized light,

$$I = \frac{I_0}{2}$$

(ii) For plane polarized light,

$$I_T = I \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$

Solution: $I_T = \frac{I_0}{2} \cos^2 60°$

$$I_T = 0.125 \, I_0$$

∴ The percentage of incident unpolarized light transmitted through the system is

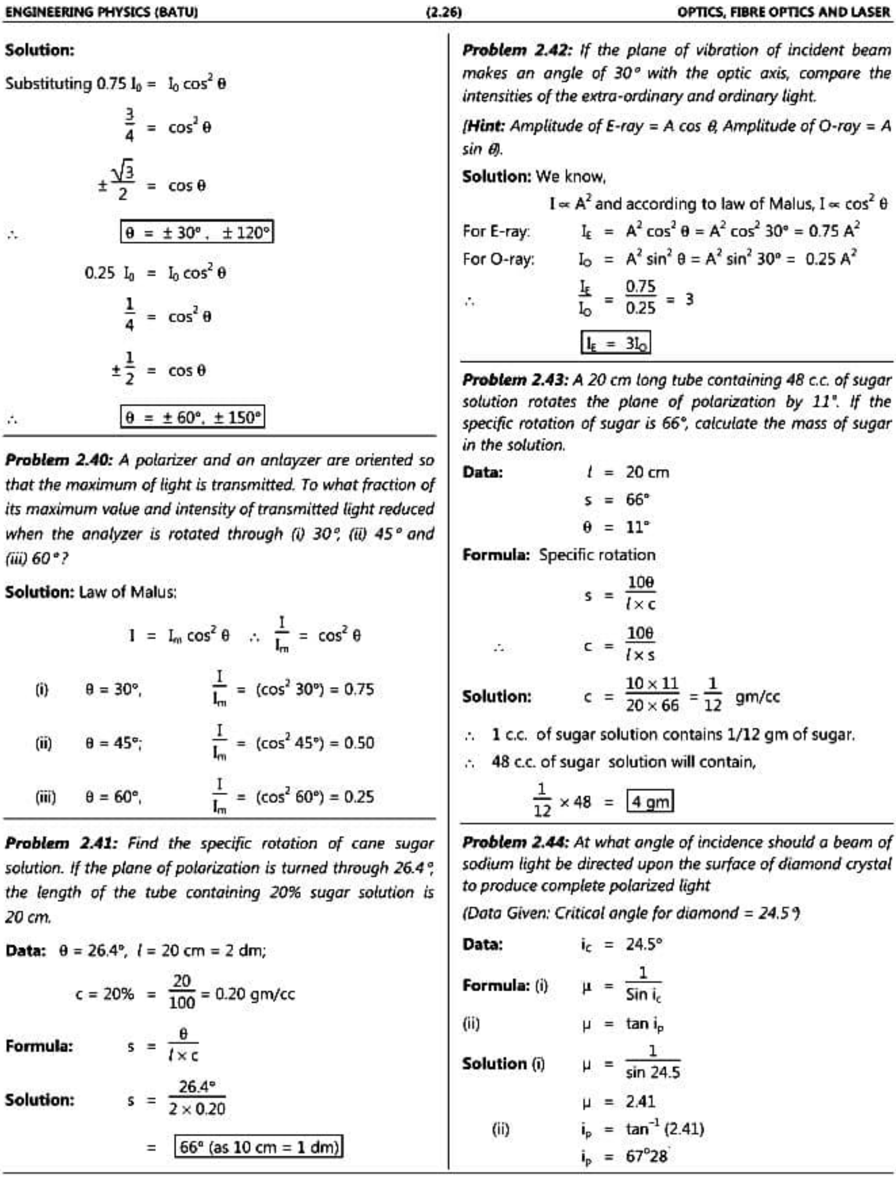
$$\% I_T = 0.125 \times 100$$

∴ $\boxed{\% I_T = 12.5\%}$

Problem 2.39: *A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. How can the analyzer be oriented so that the transmitted light is reduced to (1) 0.75, (2) 0.25 ?*

Data: (1) $I = 0.75 \, I_0$, (2) $I = 0.25 \, I_0$

Formula: $I = I_o \cos^2 \theta$



Solution:

Substituting $0.75\ I_0 = I_0 \cos^2\theta$

$$\frac{3}{4} = \cos^2\theta$$

$$\pm\frac{\sqrt{3}}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 30°,\ \pm 120°}$$

$$0.25\ I_0 = I_0 \cos^2\theta$$

$$\frac{1}{4} = \cos^2\theta$$

$$\pm\frac{1}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 60°,\ \pm 150°}$$

***Problem 2.40:** A polarizer and an anlayzer are oriented so that the maximum of light is transmitted. To what fraction of its maximum value and intensity of transmitted light reduced when the analyzer is rotated through (i) 30°, (ii) 45° and (iii) 60°?*

Solution: Law of Malus:

$$I = I_m \cos^2\theta \quad \therefore \quad \frac{I}{I_m} = \cos^2\theta$$

(i) $\theta = 30°$, $\frac{I}{I_m} = (\cos^2 30°) = 0.75$

(ii) $\theta = 45°$; $\frac{I}{I_m} = (\cos^2 45°) = 0.50$

(iii) $\theta = 60°$, $\frac{I}{I_m} = (\cos^2 60°) = 0.25$

***Problem 2.41:** Find the specific rotation of cane sugar solution. If the plane of polarization is turned through 26.4°, the length of the tube containing 20% sugar solution is 20 cm.*

Data: $\theta = 26.4°$, $l = 20$ cm $= 2$ dm;

$$c = 20\% = \frac{20}{100} = 0.20 \text{ gm/cc}$$

Formula: $s = \frac{\theta}{l \times c}$

Solution: $s = \frac{26.4°}{2 \times 0.20}$

$$= \boxed{66° \text{ (as 10 cm = 1 dm)}}$$

***Problem 2.42:** If the plane of vibration of incident beam makes an angle of 30° with the optic axis, compare the intensities of the extra-ordinary and ordinary light.*

*(**Hint:** Amplitude of E-ray = A cos θ, Amplitude of O-ray = A sin θ).*

Solution: We know,

$I \propto A^2$ and according to law of Malus, $I \propto \cos^2\theta$

For E-ray: $I_E = A^2\cos^2\theta = A^2\cos^2 30° = 0.75\ A^2$

For O-ray: $I_O = A^2\sin^2\theta = A^2\sin^2 30° = 0.25\ A^2$

$$\therefore \quad \frac{I_E}{I_O} = \frac{0.75}{0.25} = 3$$

$$\boxed{I_E = 3I_O}$$

***Problem 2.43:** A 20 cm long tube containing 48 c.c. of sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the mass of sugar in the solution.*

Data:

$l = 20$ cm

$s = 66°$

$\theta = 11°$

Formula: Specific rotation

$$s = \frac{10\theta}{l \times c}$$

$$\therefore \quad c = \frac{10\theta}{l \times s}$$

Solution: $c = \frac{10 \times 11}{20 \times 66} = \frac{1}{12}$ gm/cc

∴ 1 c.c. of sugar solution contains 1/12 gm of sugar.

∴ 48 c.c. of sugar solution will contain,

$$\frac{1}{12} \times 48 = \boxed{4 \text{ gm}}$$

***Problem 2.44:** At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light*

(Data Given: Critical angle for diamond = 24.5°)

Data: $i_c = 24.5°$

Formula: (i) $\mu = \frac{1}{\text{Sin } i_c}$

(ii) $\mu = \tan i_p$

Solution (i) $\mu = \frac{1}{\sin 24.5}$

$\mu = 2.41$

(ii) $i_p = \tan^{-1}(2.41)$

$i_p = 67°28'$

Problem 2.45: *A 20 cm long tube containing 48 c.c. of sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the mass of sugar in the solution.*

Data: $l = 20$ cm

$s = 66°$

$\theta = 11°$

Formula: Specific rotation

$$s = \frac{10\theta}{l \times c}$$

$$\therefore \quad c = \frac{10\theta}{l \times s}$$

Solution: $c = \dfrac{10 \times 11}{20 \times 66} = \dfrac{1}{12}$ gm/cc

$\therefore$ 1 c.c. of sugar solution contains 1/12 gm of sugar.

$\therefore$ 48 c.c. of sugar solution will contain,

$$\frac{1}{12} \times 48 = \boxed{4 \text{ gm}}$$

Problem 2.46: *At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light*

(Data Given: Critical angle for diamond = 24.5°)

Data: $i_c = 24.5°$

Formula: (i) $\mu = \dfrac{1}{\text{Sin} i_c}$

(ii) $\mu = \tan i_p$

Solution: (i) $\mu = \dfrac{1}{\text{Sin } 24.5}$

$\mu = 2.41$

(ii) $i_p = \tan^{-1}(2.41)$

$i_p = 67°28'$

2.15 INTRODUCTION TO LASER

- The term laser stands for **Light Amplification by Stimulated Emission of Radiation.**
- Laser is a light source which is highly coherent i.e. radiation emitted by all the emitters (atoms or molecules) in source agree in phase, direction of emission, polarisation and are essentially of one wavelength or colour (monochromatic).
- Due to coherence, a beam of laser light can travel many miles with only a negligible divergence. This makes it different from the conventional light sources which emit many wavelengths with phase and direction widely varying.
- Around 1917, Einstein first predicted the existence of two different kinds of processes by which an atom can emit radiation by (i) Spontaneous emission, (ii) Stimulated emission.
- In a laser, the process of stimulated emission is used for amplifying the light waves. The fact that stimulated emission process could be used in the construction of coherent optical sources was first put forward by Townes and Schawlow.
- The energy of an atom in any atomic system can change by
  - Absorption
  - Spontaneous emission
  - Stimulated emission.

2.16 PRINCIPLE OF LASER

2.16.1 Stimulated Emission

[Dec. 17]

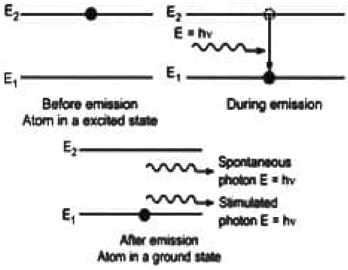


Fig. 2.31: Stimulated emission

- Consider Fig. 2.31 where the electrons are initially in the excited energy level and emission is stimulated before the spontaneous emission occurs. The excited atom is stimulated by a photon of exactly the same energy as the photon to be emitted. In such a case, two photons are emitted, one by the stimulated emission and the other stimulating photon.
- Both the photons travel in the same direction, have the same frequency and are in phase i.e. they are coherent.
- The emission of two photons with an input of only one photon implies amplification. The occurrence of spontaneous emission is directly proportional to the number of atoms in the specified energy level, whereas in stimulated emission, the rate of occurrence is proportional not only to the number of atoms in the excited state but also to the number of incident stimulating photons.

2.16.2 Population Inversion

[Dec. 17]

- The process of getting a large percentage of atoms into an excited state is called as **Population Inversion**. If a large number of atoms can be excited to upper energy levels, then the probability of stimulated emission and hence light amplification becomes greater.
- The states of the system, in which the population of the higher energy state is more than the population of the lower energy state, are called as **Negative Temperature States** (negative indicates a non-equilibrium state, not the physical state of the system).
- In any atomic system, the number of particles in a higher energy state is normally less than the number of particles in a lower energy state. If N_2 denotes the number of particles in higher energy level E_2, and N_1 denotes the number of particles in lower energy level E_1, then $N_2 < N_1$ i.e. the population of higher energy level is less than the population of lower energy level. This means that under normal conditions, the ground state E_1 is heavily populated than the excited state E_2.
- If photons of energy $h\upsilon = E_2 - E_1$ are incident on the atoms, a few of the incident photons get absorbed and some of the atoms get excited to the state E_2. This process of stimulated absorption depopulates level E_1. The rate at which this process occurs is expressed as

$$R_{12} = P_a N_1 \qquad \text{... (2.55)}$$

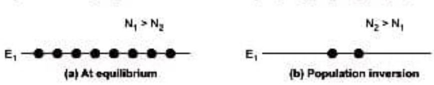
  where P_a is the probability of stimulated absorption and N_1 is the population of state E_1.
- Similarly, the stimulated emission depopulates energy level E_2 resulting in the emission of photons. The rate at which this process occurs is expressed as

$$R_{21} = P_e N_2 \qquad \text{... (2.56)}$$

  where P_e is the probability of the process of stimulated emission and N_2 is the population of state E_2.
- At thermal equilibrium, these probabilities are equal i.e. $P_a = P_e$. Then, on comparing the two rates, it is observed that more energy is absorbed than emitted.

  i.e. from (2.55) and (2.56),

  $P_a N_1 > P_e N_2$ because $N_1 > N_2$.



(a) At equilibrium (b) Population inversion

Fig. 2.32

- To produce more emission, it is essential to have $N_2 > N_1$ i.e. the number of particles in higher energy level must be made more than the number of particles in lower energy level. This is called as **Population Inversion.**
- If this inversion is achieved, there can be more emission and incoming light will be amplified coherently. A system in which population inversion is achieved is called an **Active System**.
- The method of raising atoms from lower energy levels to higher energy levels is called as **Pumping**. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light, etc. A more common method of pumping is **Optical Pumping**.

2.16.3 Metastable State

- The electron in an excited state has certain probability to decay or jump to a lower energy level. Generally, these probabilities are such that the jump occurs within 10^{-8} sec of excitation.
- However, there are some excited states, called **Metastable States**, which have a very low probability of decay i.e. electrons stay for longer time.
- Electrons may stay in the metastable excited states for seconds, minutes or even hours. In stimulated emission, the electrons must remain in excited level and wait for stimulating photon.
- Therefore, the active medium must have a metastable state. The population inversion can be obtained by using metastable states as the electrons rest in metastable state for long time.

2.16.4 Active Medium

- A medium in which the population inversion takes place is called the **Active Medium**. The active medium is responsible for the light amplification and hence LASER. The active medium may be a solid, liquid or gas and accordingly the lasers are classified as solid state or gas lasers.
- Out of the total active medium, only small number of atoms are responsible for lasing action and remaining atoms help only in hosting active atoms or in population inversion. The atoms which particulate in stimulated emission are called **Active Centres**.

2.16.5 Resonant Cavity

- A cavity can be constructed using mirrors such that the light rays return to their original position after travelling through the cavity for a certain number of times. Such cavities are known as **Resonant Cavities**.
- Fig. 2.33 shows cavity formed by two parallel mirrors M_1 and M_2. One of the mirrors is completely silvered (M_1) and the other is partially silvered (M_2). The laser beam emerges from the resonant cavity through the partially silvered mirror M_2.

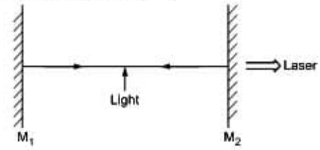


Fig. 2.33: Resonant cavity

- The active system is placed in the resonant cavity, the photon emitted will keep on reflecting back and forth within the cavity. The light which is incident parallel to the axis of optical cavity will only leak out as a laser. That is why, laser is highly directional.

2.16.6 Pumping

The method of raising atoms from lower energy levels to higher energy levels is called as **Pumping**. The pumping is used for achieving population inversion which is necessary for optical amplification to take place. There are several methods for pumping electrons. They are as follows:

1. **Optical Pumping**

- In optical pumping, an external light source (flash lamp) is used to produce a high population in some particular energy level E_2 (say) by selective absorption as shown in Fig. 2.34.

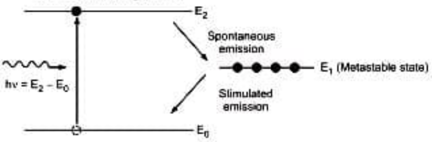


Fig. 2.34: Optical pumping

- When a flash of light falls on electrons in ground state, they absorb incident photons and get excited. After staying there for some time, some of the atoms make spontaneous transition to metastable state E_1. As the probability of spontaneous decay is less in metastable state, a large population accumulates in this level. This results in a population inversion between E_0 and E_1.
- Generally, this method is used in solid-state lasers, such as ruby laser.

2. **Inelastic Atom-Atom Collisions**

- Here suitable mixtures of gases are used. The gases are selected in such a way that their excited states are almost same. This makes the energy exchange possible between the atoms of the gases. If two gases A and B have same excited state, A^* and B^* then,

$$A^* + B \rightarrow A + B^*$$

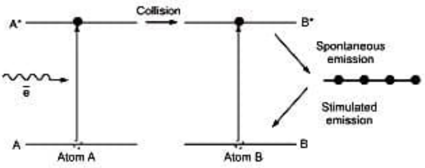


Fig. 2.35: Inelastic atom-atom collision

- The atom of gas A is excited by electric discharge. In collision with B, the energy is transferred to B. As a result, the excited level of atom B becomes more populated than lower level to which B can decay, as shown in Fig. 2.35.
- The best example is the He-Ne gas laser.

3. **Forward Biasing of a p-n Junction**

- If a p-n junction is formed with degenerate (heavily doped) semiconductors, the bands under forward bias appear as shown in Fig. 2.36.

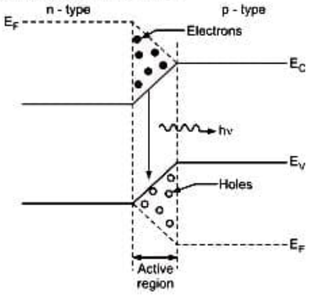


Fig. 2.36: Forward biasing of a p-n junction diode

- If the bias voltage is large enough, electrons and holes are injected into the active region. As a result, the depletion layer now contains a large number of electrons in conduction band and holes in valence band. If the population density is high enough, it gives population inversion. For a population inversion, the applied voltage should be selected in such a way that $eV > h\upsilon$ ($= E_g$).

- The other methods of pumping are electron excitation, chemical reactions, etc. These methods will not be discussed in detail as they are beyond the scope of the text.

2.17 EINSTEIN'S COEFFICIENTS

Consider an assembly of atoms at thermal equilibrium. The system is at temperature T with radiation of frequency υ and energy density I_υ (or photon density). Let N_1 and N_2 be the number of atoms in energy states E_1 and E_2 respectively at any time t.

The rate of transition from E_1 to E_2 will depend on the properties of energy states E_1 and E_2 and is proportional to the energy density I_υ of the radiation of frequency υ and the number of electrons N_1 in energy state E_1.

$$\text{Therefore,} \quad R_{12} \propto N_1 I_\upsilon$$

$$\text{or} \quad R_{12} = B_{12} N_1 I_\upsilon \qquad \ldots (2.57)$$

where B_{12} is the proportionality constant called **Einstein's Coefficient for Absorption.**

The electron in the excited level will make transition to a lower energy state either by spontaneous or stimulated emission.

In spontaneous emission, the rate of emission is proportional to the number of excited electron N_2.

$$\text{i.e.} \quad (R_{21})_{\text{spont}} \propto N_2$$

$$\therefore \quad (R_{21})_{\text{spont}} = A_{21} N_2 \qquad \ldots (2.58)$$

where A_{21} is called the Einstein's coefficient of spontaneous emission.

But in stimulated emission the rate of emission will depend upon number of excited electron N_2 and intensity of the stimulating photons I_υ.

$$\therefore \ (R_{21})_{\text{stimulated}} \propto N_2 I_\upsilon$$

$$\text{or } (R_{21})_{\text{stimulated}} = B_{21} N_2 I_\upsilon \qquad \ldots (2.59)$$

where B_{21} is called the Einstein's coefficient of stimulated emission.

At equilibrium the rate of absorption and emission is same.

$$\therefore \quad R_{21} = (R_{21})_{\text{spont}} + (R_{21})_{\text{stimulated}} \qquad \ldots (2.60)$$

From equations (2.57), (2.58) and (2.59),

$$B_{12} N_1 I_\upsilon = A_{21} N_2 + B_{21} N_2 I_\upsilon$$

$$I_\upsilon (N_1 B_{12} - N_2 B_{21}) = A_{21} N_2$$

$$I_\upsilon = \frac{A_{21} N_2}{N_1 B_{12} - N_2 B_{21}} \qquad \ldots (2.61)$$

Dividing by $N_2 B_{21}$,

$$I_\upsilon = \frac{A_{21}/B_{21}}{\left(\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right)} \qquad \ldots (2.62)$$

From Boltzmann distribution law,

$$N_1 = N_0^{-E_1/kT} \qquad \ldots (2.63)$$

$$\text{and} \quad N_2 = N_0 e^{-E_2/kT} \qquad \ldots (2.64)$$

where N_0 = total electrons in ground state

k = Boltzmann's constant

$$\therefore \quad \frac{N_1}{N_2} = e^{-E_1 + E_2/kT}$$

$$\text{But} \quad E_2 - E_1 = h\upsilon$$

$$\therefore \quad \frac{N_1}{N_2} = e^{h\upsilon/kT} \qquad \ldots (2.65)$$

Substituting in equation (2.62),

$$I_\upsilon = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} e^{h\upsilon/kT} - 1\right)} \qquad \ldots (2.66)$$

The Planck's radiation formula is given by

$$I_\upsilon = \frac{8\pi h\upsilon^3}{c^3} \left(\frac{1}{e^{h\upsilon/kT} - 1}\right) \qquad \ldots (2.67)$$

Comparing equations (2.66) and (2.67), we have

$$B_{12}/B_{21} = 1 \quad \text{i.e.} \quad B_{12} = B_{21} \qquad \ldots (2.68)$$

$$\text{and} \quad \frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3} \qquad \ldots (2.69)$$

The equations (2.68) and (2.69) are called **Einstein's Relations**. The ratio of spontaneous to stimulated emission is proportional to υ^3.

From equations (2.58) and (2.59), the ratio of spontaneous emission to stimulated emission is

$$R = \frac{A_{21} N_2}{B_{21} N_2 I_\upsilon} = \frac{A_{21}}{B_{21} I_\upsilon} \qquad \ldots (2.70)$$

Using equations (10) and (14)

$$R = \frac{A_{21}/B_{21}}{A_{21}/B_{21}} (e^{h\upsilon/kT} - 1)$$

$$\therefore \quad R = e^{h\upsilon/kT} - 1 \qquad \ldots (2.71)$$

Therefore, at thermal equilibrium at temperature T for $\upsilon << \frac{kT}{h}$, the number of stimulated emission exceeds the spontaneous emission, while for $\upsilon >> \frac{kT}{h}$, the number of spontaneous emission exceeds the number of stimulated emission.

2.18TYPES OF LASER

- Depending upon the energy levels involved in the pumping the lasers can be classified in following categories.
  - (a) Two level.
  - (b) Three level.
  - (c) Four level.

(a) Two Level Laser System

- A two-level laser system consists of only two energy levels, E_1 and E_2, ground state and excited state. The electrons from E_1 are pumped to E_2 as shown in Fig. 2.37.

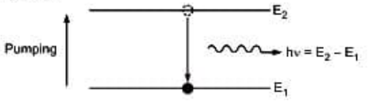


Fig. 2.37: Two-level laser system

- The electron from E_2 decays to E_1 radiating a photon of energy $h\upsilon$. The best example of two-level laser system is a diode laser.

(b) Three Level Laser System

- In a three-level laser system, three energy levels E_1, E_2 and E_3 are involved as shown in Fig. 2.38. Here one of the transitions is non-radiative. The transition between E_3 to E_2 is very fast and non-radiative.
- Here the electron is pumped to E_3 directly. As decay from E_3 to E_2 is very fast, hence E_2 will be more populated and decay from E_2 to E_1 gives a photon of energy $h\upsilon$. The best example of this category is ruby laser.

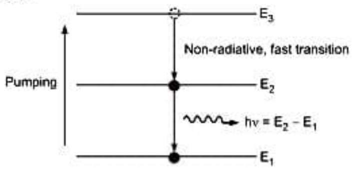


Fig. 2.38: Three-level laser system

(c) Four Level Laser System

- In a four-level laser system, the electrons are pumped to E_4 directly. The transitions will take place between E_4 to E_3, E_3 to E_2 and E_2 to E_1. Out of these three transitions, only one will be radiative and two will be non-radiative.
- Fig. 2.39 shows a four-level laser system. The best example is He-Ne laser. In a four level laser system, lasing action is always observed between E_3 and E_2.
- The life time of E_4 is very short. The transition between E_2 to E_1 is non-radiative and spontaneous. Moment electron reaches to E_1, it is pumped to level E_4, but due to short life time of energy level E_4, electron immediately jumps to E_3. Hence the levels E_1 and E_4 are free to accommodate electrons and the population of E_3 is always higher which favours for lasing action. Therefore, four level laser system works effectively.

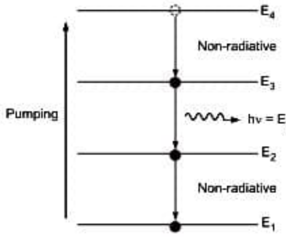


Fig. 2.39: Four-level laser system

Need of Three/Four-Level System

- If there are only two states, ground state and metastable state. When a photon is incident on it, it will be absorbed and electron will jump from ground state to metastable state. At the same time, due to stimulated emission, electron will jump to the ground state.
- During the process a situation will arise when half of the atoms are in the ground state (N_1) and half in the metastable state (N_2). i.e. $N_1 = N_2$. This will make the rate of stimulated emission and absorption equal. But to achieve population inversion, rate of absorption should be higher than stimulated emission.
- Thus if there are only two states the population inversion could not be achieved. And, therefore, laser action will not be possible.

2.19 RUBY LASER – THREE LEVEL LASER SYSTEM [Dec. 18, May 19]

A ruby laser is a solid-state laser that uses a synthetic ruby crystal. Typical ruby laser is a pulsed laser of intense red colour.

Construction

- The laser consists of a ruby rod surrounded by a flash tube. One end of the rod is highly silvered while the other end is semi-silvered. The flash tube surrounds the ruby rod in the form of a spiral.

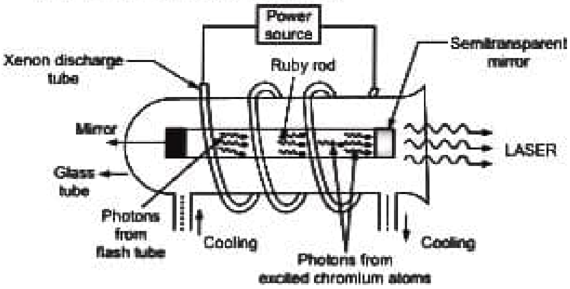
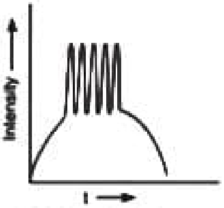


Fig. 2.40

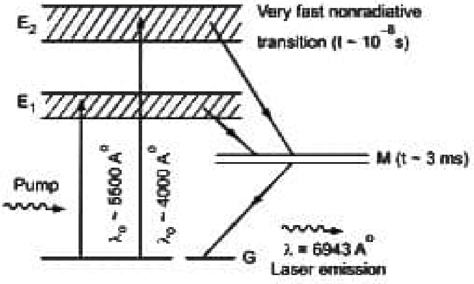
- Synthetic ruby consists of a crystal of aluminum oxide (Al_2O_3) in which a few of the aluminum atoms (Al^{3+}) are replaced by chromium atoms (Cr^{3+}). These atoms have the property of absorbing green light.
- The chromium impurity is the active atom of the laser. Doping of chromium gives ruby its characteristic red colour.

(a) Pumping and Energy Levels of Chromium

- When ruby is in a steady magnetic field, chromium acquires energy states, of which three are represented schematically as shown in Fig. 2.41 (a).
- As is clear from the figure, this is a three-level laser system. Level M actually consists of a pair of levels corresponding to wavelengths of 6943 A° and 6929 A°. However, laser action takes place only on 6943 A° line due to higher population inversion.
- The pumping of chromium atoms is performed with a Xenon or Krypton flash lamp. The chromium atoms in the ground state absorb radiation around wavelengths 5500 A° and 4000 A° and are excited to the levels marked E_1 and E_2.



(a) Ruby-pulsed laser



(b) Energy level

Fig. 2.41

(b) Assembly of Chromium Atoms to Metastable State

- The chromium atoms excited to these levels, relax rapidly through a non-radiative transition (in a time 10^{-8} to 10^{-9} sec) to the metastable state M, which has a life time of ~3 m secs. Laser emission occurs between level M and the ground state G at an output wavelength of 6943 A°.

(c) Operation

- The operational sequence starts with the ignition of the Xenon flash tube. Chromium atoms in the ruby rod are energized by absorption of the energetic photons from the flash tube.
- When the excited electrons in the chromium atoms fall back to their normal states, photons are given off by spontaneous emission emitting red light (hence ruby has a natural red colour). Some of these photons escape from the rod but many oscillate or bounce back and forth along the length of the rod with the help of the mirror at the two ends.
- When the electrons in the excited state are exposed to these radiations of the same frequency which they are about to emit, the emission process is triggered. Radiation is now emitted, which is exactly in phase with the exposed radiation.
- This cumulative process of flash tube photons exciting chromium atoms which in turn emit photons in the same direction and phase, continues until the coherent laser beam penetrates through the partially reflecting mirror on one end of the rod to give a powerful beam of red light.

(d) Pulsed Output

- A certain stage is reached when the population inversion caused by one flash of Xenon tube is used up. As soon as the flash lamp stops operating, the population of the upper level is depleted very rapidly and laser action ceases until the arrival of the next flash. Refer Fig. 2.41 (b).
- Thus, ruby is a **Pulsed Laser**. The output beam has a principal wavelength of 6943 A° equal to 4.3×10^{14} Hz frequency (lies in the visible spectrum). The duration of the output flash is about 300 μsec.
- During the operation of a ruby laser, a very high temperature is produced. To prevent any damage to the ruby rod, it is surrounded by a liquid nitrogen container and is operated to give out the beam only in pulses.
- This laser is used in many applications as its output lies in the visible region where photographic emulsions and photo detectors are more sensitive than they are in the infrared region. Ruby lasers also find application in laser holography, laser ranging, etc.

2.20 HELIUM-NEON LASER – FOUR LEVEL LASER SYSTEM [May 18]

- This is a **'Continuous Laser'** unlike the ruby laser. In this laser, the vapours of metal are used as the media.
- It is an extremely popular form of laser as it is simple, inexpensive and has an extremely broad range of emission wavelengths (0.6 to 100 μm depending on the type of gas used). The first gas laser to be operated successfully was the He - Ne laser.
- In solid-state lasers, pumping is usually done by using a flash lamp or a continuous high power lamp. Such a technique is efficient if the laser system has broad absorption bands. In gas lasers, as the atoms are characterized by sharp energy levels, an electrical discharge is generally used to pump the atoms.

Construction

- It consists of a quartz tube with a diameter of about 2-8 mm and a length of 10-100 cm. It is filled with helium and neon. The pressure of helium is approximately 10 times that of neon.
- The neon atoms provide energy states for the transitions while helium provides a mechanism for efficiently exciting neon atoms to upper metastable states i.e. helium serves merely as an energy transfer agent.
- At one end of the tube is a total reflector while at the other is a partial reflector. The gas is excited by means of a high frequency generator.

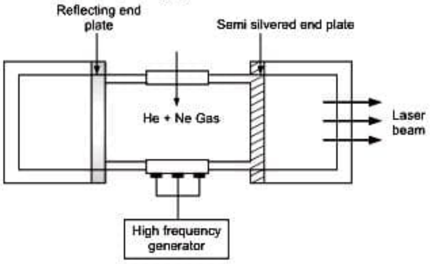


Fig. 2.42: He-ne laser

Principle

- In He-Ne laser, population inversion is produced through inelastic collisions between excited He atoms and Ne atoms in the ground state. The process can be expressed as

  $He^* + Ne \rightarrow He + Ne^*$ (* shows an excited state)

- This is possible, because the levels Ne_4 and Ne_6 of neon atoms have almost the same energy as the levels He_2 and He_3 of helium atoms as shown in the energy level diagram.

Working

(a) Electric Discharge and Excitation of Helium

- When an electrical discharge is passed through the gas, the electrons which are accelerated down the tube collide with helium and neon atoms and excite them to higher energy levels.
- The helium atoms tend to accumulate at the levels He_2 and He_3 due to their long life times of $= 10^{-4}$ secs and 10^{-6} secs respectively.

(b) Transfer of Energy from Helium to Neon and Pumping

- As the levels Ne_4 and Ne_6 of neon atoms have almost the same energy as He_2 and He_3, excited helium atoms colliding with neon atoms in the ground state can excite the neon atoms to Ne_4 and Ne_6 states.
- As the pressure of helium is ten times that of neon, the levels Ne_4 and Ne_6 of neon are selectively populated as compared to other levels of neon.

  i.e. $He^* + Ne \rightarrow He + Ne^*$ (* indicates excited state)

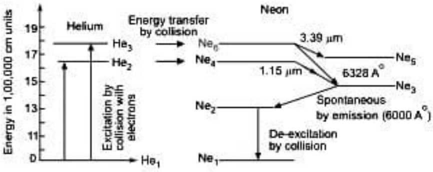


Fig. 2.43

(c) Population Inversion for Neon

- Transition between Ne_6 and Ne_3 produces the popular 6328 A° (632.8 nm) line of He - Ne laser. Neon atoms de-excite through spontaneous emission from Ne_3 to Ne_2 (life time $\sim 10^{-8}$ sec.). As this time is shorter than the life time of level Ne_6 ($\sim 10^{-7}$ sec.), steady state population inversion can be achieved between Ne_6 and Ne_3. Level Ne_2 is metastable and thus tends to collect atoms.
- The atoms from this level fall back to the ground level mainly through collisions with the walls of the tube. As Ne_2 is metastable, it is possible for the atoms in this level to absorb the spontaneously emitted radiation in $Ne_3 \rightarrow Ne_2$ transition to be re-excited to Ne_3. This tends to reduce the effect of inversion.
- It is for this reason that the gain in this laser transition is found to increase with decreasing tube diameter.

(d) Transition within Neon and Continuous Output

- The other two important wavelengths from the He - Ne laser correspond to the $Ne_4 \rightarrow Ne_3$ (1.15 μm) and $Ne_6 \rightarrow Ne_5$ (3.39 μm) transitions. The laser can be made to oscillate at 6328 A° by using optical elements (multilayer coated mirrors) in the path. These lasers are continuous, because the collision process maintains the energy states Ne_6 and Ne_4 at larger population densities than the lower states. This continued population inversion gives a continuous lasing action.
- A typical He - Ne laser operates with a current of 10 mA at a D.C. voltage of 2500 V and gives an optical output of 5 mW. Its efficiency is then $\frac{5 \times 10^{-3}}{2500 \times 10^{-2}}$ = 0.02 %.
- This is the only laser radiating in far infrared region. Hence, mostly used in laser '**Raman Spectroscopy**'.

2.21 APPLICATIONS OF LASER

2.21.1 Applications of Laser in Industry

- Laser can be focused to a very high energy density into a small image (≈ 1 micron in diameter) with the help of suitable lenses. Due to the small size of the image and the control over the energy, lasers are used extensively for cutting, welding and drilling circuits.
- **Drilling:** A laser beam is also used to drill holes of micron dimensions on printed circuit boards (PCBs). It is also used in resistance trimming in electric components industries. One can drill holes of the diameter of 10 μm through very hard substances like diamond. YAG laser is found to be very useful in such applications.
- **Welding:** Lasers are used as a heat source in welding the joints of the metals. This type of precise welding is extremely important in micro-electronics in which thin films are used. Thermocouple wires can easily be welded with the help of high power laser beam.
- **Micromachining:** Lasers are used for machining a surface in a slow and accurate manner to achieve an extraordinarily smooth finish.
- **Cutting:** Another important industrial application is metal or fabric cutting. A finely focused laser beam can cut thick and hard metal sheets with high precision and accuracy. It is also used in tailoring industries to cut thousands of layers of cloth at one instant.
- Due to its intensity and directionality, laser is used in surveying. When tunnels are to be constructed, engineers use the laser beam as a reference, to check that it is being constructed along a straight line. Similarly, it can be used to dig a ditch to a certain prescribed depth. Its most interesting use in surveying has been in measuring the distance from the earth to the moon. This distance was measured to an accuracy of 600 ft, and with the aid of reflectors to within six inches. This accuracy will allow to determine the location of the north pole to within six inches. It is further believed that a laser could be used to check whether the gravitational constant is actually a constant.
- A laser beam can determine precisely the distance, velocity and direction as well as the size and form of distant objects by means of the reflected signal as in radar. A Lidar (Laser radar), which sends out beams of laser light and detects echoes even from atmospheric layers has been developed.

2.21.2 Applications of Laser in Medicine

- Bloodless cancer surgeries can be performed as the beam can be focused on a small area, so that only the harmful tissue can be destroyed without damaging the surrounding region.
- Laser has been successfully used in ophthalmology, in the treatment of detached retinas, in welding cornea, etc. At the command of the physician, laser produces a beam of light which is directed onto the eye under treatment, to produce a minute coagulation. A series of these lesions weld the detached retina.
- Laser is used as a tool in the study of genetics. Lasers have been built into or are devised to be attached to microscopes. As high density energy is achieved, it can be used in micro-surgery, micro-burning, etc. Such a microscopic laser can concentrate millions of watts of power per square millimeter into a selected area. For example, a focused microscope laser can be used to make tiny openings (of 25 μ in diameter) in the cell walls, of say the nervous system, heart, retina, etc. without causing irreversible damage.
- Laser microprobes can be used as dental drills giving an advantage of no heating, no anesthetic and no pain to the patient. They have also been successfully used for localized treatment of skin growths and blemishes in human beings. A large amount of energy can be transmitted through the skin to interact with deeper different biological materials or structures which are damaged.

2.21.3 Application of Laser in Communication

- In this technology, optical energy is transferred through a guided media, called the **Glass Fibre**. When a beam of light enters at one end of a transparent rod (glass rod say), the light beam is totally internally reflected and gets trapped within the rod.
- A similar behaviour is exhibited by a bundle of fine fibres. A beam enters at one end and is transmitted through the wire to the other end, even when the fibre is curved.
- One of the most important areas of application of fibre optics is in telecommunication. The communication kit consists of a transmitter, optical fibre and the receiver. The block diagram is as shown in Fig. 2.44.

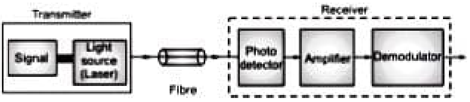


Fig. 2.44: Communication kit

- The transmitter consists of a light source, either LED or laser diode, with a signal. The light beam from the source is connected to the fibre, through optical connections.
- The carrier and signal frequency propagate through the fibre.
- At the other end, it is detected with the help of a photodetector. The received signal is demodulated and the information is stored or displayed by electronic circuits.

2.21.4 Application in Information Technology (Holography)

Holography

- This is a technique of producing an interference pattern between a direct laser beam and a laser beam reflected from an object on a photographic plate. This pattern on the developed photographic plate, when illuminated with laser in a proper manner, produces a three-dimensional image of the object called a '**Hologram**'.
- Holography deals with three-dimensional image of the object whereas photography is a two-dimensional effect. In photography, the photographic plate records only the intensity of light due to the image formed on it. In holography, both the intensity and phase distribution are recorded simultaneously using interference technique. Due to this the image produced by the technique of holography has a true three-dimensional form and is as true as the object

2.22 OPTICAL FIBRE

2.22.1 Principle of Optical Fibre

- Optical fibre is a very thin and flexible medium having a cylindrical shape consisting of three sections: (i) The core, (ii) The cladding and (iii) The outer jacket.

Principle of Light Transmission

- The principle of light transmission through optical fibre is total internal reflection. For total internal reflection to take place at the fibre wall, the following conditions should be satisfied:
  - ➢ The refractive index of the core material (μ_1) must be greater than that of the cladding (μ_2).

- At the core-cladding interface, the angle of incidence θ must be greater than the critical angle,

  where $\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$.

- When a light ray travels from a denser to a rarer medium, the angle of refraction is greater than the angle of incidence. As the angle of incidence increases, the angle of refraction also increases and for a particular angle of incidence, the refracted ray grazes the interface between the core and the cladding. This angle of incidence is called as the **Critical Angle θ_c**.
- If angle of incidence is greater than θ_c, the ray will be reflected back into the core, i.e. it suffers **Total Internal Reflection**. For angles equal to or greater than the critical angle the light will be totally reflected and no light will be refracted. Fig. 2.45 shows total internal reflection.
- When light is incident on core of the fibre optics, it will be refracted and will travel in the core. After some time it will strike one of the core-cladding interface say upper surface. If the angle of incidence is greater than critical angle, it will be totally reflected and remain in the core.
- Now, the reflected light will travel to the lower surface. It is then incident on the lower surface where the same process is repeated and light gets transmitted from one end to the other end as shown in Fig. 2.45.

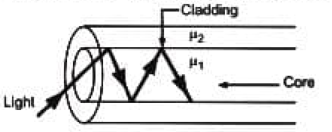


Fig. 2.45: Propagation of light in fibre optics

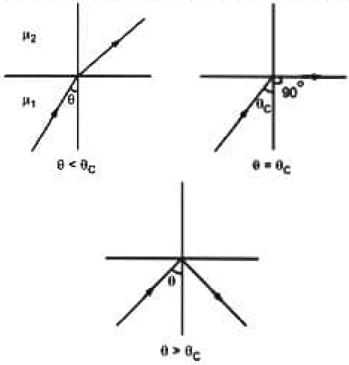


Fig. 2.46: Total internal reflection

2.22.2 Application of Fibre Optics in Communication Kit

- One of the most important areas of application of fibre optics is in telecommunication. The communication kit consists of a transmitter, optical fibre and the receiver. The block diagram is as shown in Fig. 2.47.

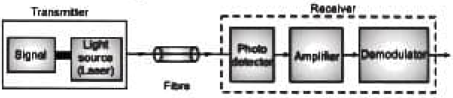


Fig. 2.47: Communication kit

- The transmitter consists of a light source, either LED or laser diode, with a signal. The light beam from the source is connected to the fibre, through optical connections.
- The carrier and signal frequency propagate through the fibre.
- At the other end, it is detected with the help of a photodetector. The received signal is demodulated and the information is stored or displayed by electronic circuits.

2.22.3 Introduction to Optical Fibre

- Efforts to device communication systems for sending from one place to another distant place have been continuing by the human being. These systems have used as optical or acoustical means like signal lamps or horns, electrical codes like Morse Code (1938), Telephones (1878) so effort. A portion of electromagnetic waves is used in telephones, amplitude modulated and frequency modulated radio, television, CB (citizen's band radio), satellite link in the recent days.
- Optical region of the electromagnetic spectrum which contains wavelengths from 50 nm (500 A°) ultraviolet) to about 1000 nm (10000 A°) (far infrared) with the visible range from 400 nm to 700 nm (nanometre, 10^{-9} m).
- Advances in technology have made it possible to use optical fibres alongwith good optical sources, photodetectors and fibre cable connectors to transmit more data at high transmission rate from one place to a distant place.

2.22.4 Structure of Optical Fibre

- Optical fibre is a dielectric waveguide and it operates at optical frequencies (5 × 10^4 Hz). It is generally cylindrical, the core of which has higher refractive index (n_1) than that of the surrounding material (n_2). The core and the surrounding dielectric together form an optical waveguide. Depending on the type of the waveguide, optical fibres are categorised into two steps as

1. Step Index Fibre and
2. Graded Index Fibre.

- The path through which light is propagated is called **Waveguide**. In case of optical fibre, core and cladding together work as optical waveguide.

1. Core-Cladding Fibre

Core : In this type of optical fibre, there is a single solid dielectric cylinder of radius a and refractive index n_1 as shown in Fig. 2.48. This solid cylinder is known as the **Core** of the optical fibre.

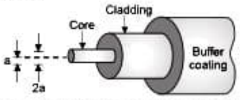


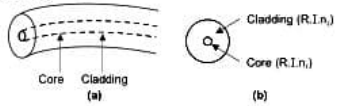
Fig. 2.48 : Construction of step index optical fibre

Cladding (as a Rarer Medium) :

- The core is surrounded by a solid dielectric (cone polymer) **Cladding** having a refractive index n_2 which is less than n_1. Cladding reduces the scattering losses due to dielectric discontinuities at the core surface.
- Also, it adds to the mechanical strength of the fibre and protects the core from absorbing the surface contamination which can come in contact with it. The cladding is made up of either glass or plastic materials.

Buffer (for Mechanical Strength) :

- Most fibres are encapsulated in an elastic abrasion resistant plastic material. This encapsulating material is called **Buffer Coating**. The buffer adds further mechanical strength to the fibre and keeps away the fibre from small geometrical irregularities, distortions or roughness of the surrounding surfaces. This also avoids random microscopic or sharp bends when the fibres are incorporated into cables or when supported on some other structures.
- The conventional optical fibre consists of a core region of refractive index, n_1, which is surrounded by a cladding of lower refractive index, n_2. The fibre or a bundle of fibres is sheathed in an outer protective covering.



(a) (b)

Fig. 2.49 : The fibre with core and cladding

There are two types of optical fibres, viz. (a) step index and (b) graded index.

2.22.5 Types of Optical Fibre

- **Step-Index Optical Fibre :** The core has an **Uniform Refractive Index**, n_1 and the cladding has an **Uniform Refractive Index** n_2 ($n_2 < n_1$). Let the core radius be 'a' and the outer radius of cladding be 'b'.

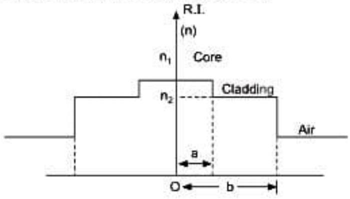


Fig. 2.50 : The refractive index profile of a step-index fibre

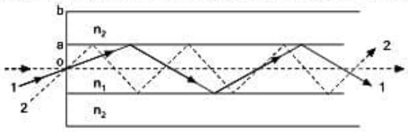


Fig. 2.51 : The paths of rays in step-index fibre

- In the step-index fibre, rays entering at different angles of incidence with the axis travel different path lengths and emerge out at different times. This results in **Pulse Dispersion**, i.e. an input pulse gets widened as it travels along the fibre.
- **Graded-Index Optical Fibre :** The **Refractive Index of the Core Varies Continuously** from n_1 at the centre to n_2 at the core-cladding interface. The cladding has the constant refractive index n_2.
- In the graded-index fibre, a ray is continuously bent and travels a periodic path along the axis. Rays entering at different angles follow different paths with the same period, both in space and time. This results into periodic **Self Focussing** of the rays as shown in Fig. 2.52. Therefore, the pulse dispersion is less as compared with the step-index fibre.

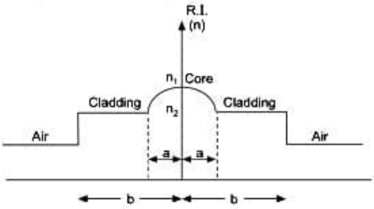


Fig. 2.52 : The R.I. profile of a graded-index fibre

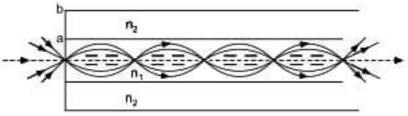


Fig. 2.53 : The paths of rays in graded-index fibre

- The core diameters (2a) in fibres in use may range between 4 μm to 100 μm. The core + cladding diameter (2b) usually ranges between 100 μm to 200 μm.
- Fibres with narrow cores (about 10 μm) allow only one wave-mode to pass and are called **Monomode Fibres;** while those with core diameters about 50 μm and above allow different wavemodes and are called **Multimode Fibres.**
- Optical fibres used commonly in telecommunication applications have 2b = 125 μm and (typical values for refractive indices are : $n_1 \equiv 1.5$ and $n_2 = n_1 (1 - \Delta)$ where Δ = 0.01 to 0.02 (1 to 2 %)).

2.22.6 Index Difference (Core-Cladding Index Difference)

- In practical step-index fibres, the core with radius has refractive index n_1. A typical value of n_1 is 1.48. The cladding surrounding this core has a refractive index slightly lower than n_1. The relation between n_1 and n_2 is given by

$$n_2 = n_1 (1 - \Delta)$$

where Δ is called core-cladding index difference or simply the index difference.

- The values of n_2 are chosen so that the index difference Δ is equal to 0.01. As the core is having higher refractive index than that of cladding, electromagnetic waves at optical frequencies propagate along the fibre due to **Total Internal Reflection** at the core-cladding interface.
- An optical fibre may be either monomode or multimode in case of both the types i.e. step index and graded-index fibre.

2.22.7 Advantages of Optical Fibres

- Optical frequencies are extremely large. Since the frequency of light used as carrier is of the order of 10^{15} Hz, the information carrying capacity of a fibre is much greater.
- The optical fibres are made of dielectric material, which offers electrical isolation between input and output parts of the circuit.
- The material used in fibres is silica glass (or SiO_2). As this is available abundantly on earth, the cost of fibre lines is much lower.
- As fibres have a high information capacity, multiple channel routes can be compressed into very small cables. This helps in reducing congestions in overcrowded cable ducts.
- As fibres are very thin, light and occupy less space, a large number of them can be used at a time. Because of this, detailed images can be obtained. This is of particular importance in medicine where an endoscope, employing fibre optics is increasingly being used to take pictures inside the human body.
- The transmission is due to internal reflection, therefore there is less loss. The cable is immune to electric, magnetic or R.F. fields in atmosphere, because it is covered with a cladding.
- Active scintillating fibres (fibre lasers) are useful in developing flexible high intensity laser probes.

2.22.8 Acceptance Angle and Numerical Aperture

[May 19]

Consider Fig. 2.54. ϕ is the angle of incidence at the core-cladding interface for a ray entering the core making an angle, i, with the fibre axis.

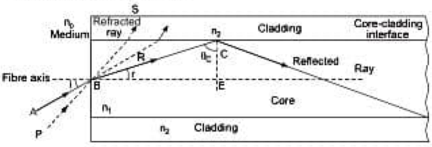


Fig. 2.54 : Ray propagation through step-index optical fibre

The condition for total internal reflection to take place is

$$\sin \phi \geq \frac{n_2}{n_1} \qquad \text{... (2.72)}$$

We have, $\sin \phi = \sin (90 - r) = \cos r$ ... (2.73)

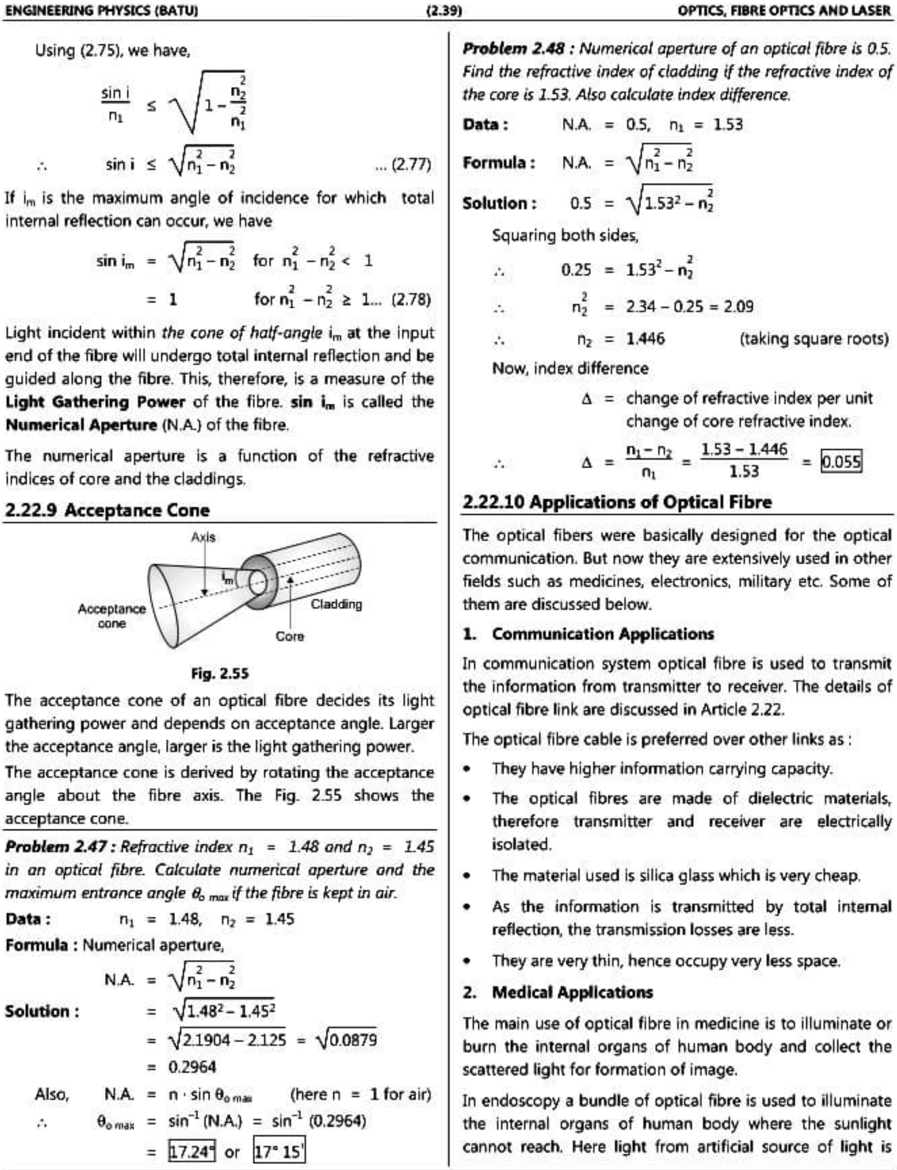
Also, $\frac{\sin i}{\sin r} = n_1, \quad \therefore \sin r = \frac{\sin i}{n_1}$ ... (2.74)

Now, $\sin r = \sqrt{1 - \cos^2 r}$

$= \sqrt{1 - \sin^2 \phi}$ ... (2.75)

The condition of total internal reflection, (2.72), therefore can be expressed as

$$\sin r \leq \sqrt{1 - \frac{n_2^2}{n_1^2}} \qquad \text{... (2.76)}$$



Using (2.75), we have,

$$\frac{\sin i}{n_1} \le \sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\therefore \quad \sin i \le \sqrt{n_1^2-n_2^2} \quad \text{... (2.77)}$$

If i_m is the maximum angle of incidence for which total internal reflection can occur, we have

$$\sin i_m = \sqrt{n_1^2-n_2^2} \quad \text{for } n_1^2-n_2^2 < 1$$

$$= 1 \quad \text{for } n_1^2-n_2^2 \ge 1 \text{... (2.78)}$$

Light incident within *the cone of half-angle* i_m at the input end of the fibre will undergo total internal reflection and be guided along the fibre. This, therefore, is a measure of the **Light Gathering Power** of the fibre. **sin i_m** is called the **Numerical Aperture** (N.A.) of the fibre.

The numerical aperture is a function of the refractive indices of core and the claddings.

2.22.9 Acceptance Cone

Fig. 2.55

The acceptance cone of an optical fibre decides its light gathering power and depends on acceptance angle. Larger the acceptance angle, larger is the light gathering power.

The acceptance cone is derived by rotating the acceptance angle about the fibre axis. The Fig. 2.55 shows the acceptance cone.

***Problem 2.47 :** Refractive index $n_1 = 1.48$ and $n_2 = 1.45$ in an optical fibre. Calculate numerical aperture and the maximum entrance angle $\theta_{o\ max}$ if the fibre is kept in air.*

Data : $n_1 = 1.48, \quad n_2 = 1.45$

Formula : Numerical aperture,

$$\text{N.A.} = \sqrt{n_1^2-n_2^2}$$

Solution :

$$= \sqrt{1.48^2-1.45^2}$$

$$= \sqrt{2.1904-2.125} = \sqrt{0.0879}$$

$$= 0.2964$$

Also, $\text{N.A.} = n \cdot \sin \theta_{o\ max}$ (here $n = 1$ for air)

$$\therefore \quad \theta_{o\ max} = \sin^{-1}(\text{N.A.}) = \sin^{-1}(0.2964)$$

$$= \boxed{17.24^\circ} \text{ or } \boxed{17^\circ\ 15'}$$

***Problem 2.48 :** Numerical aperture of an optical fibre is 0.5. Find the refractive index of cladding if the refractive index of the core is 1.53. Also calculate index difference.*

Data : $\text{N.A.} = 0.5, \quad n_1 = 1.53$

Formula : $\text{N.A.} = \sqrt{n_1^2-n_2^2}$

Solution : $0.5 = \sqrt{1.53^2-n_2^2}$

Squaring both sides,

$$\therefore \quad 0.25 = 1.53^2-n_2^2$$

$$\therefore \quad n_2^2 = 2.34-0.25 = 2.09$$

$$\therefore \quad n_2 = 1.446 \quad \text{(taking square roots)}$$

Now, index difference

Δ = change of refractive index per unit change of core refractive index.

$$\therefore \quad \Delta = \frac{n_1-n_2}{n_1} = \frac{1.53-1.446}{1.53} = \boxed{0.055}$$

2.22.10 Applications of Optical Fibre

The optical fibers were basically designed for the optical communication. But now they are extensively used in other fields such as medicines, electronics, military etc. Some of them are discussed below.

1. Communication Applications

In communication system optical fibre is used to transmit the information from transmitter to receiver. The details of optical fibre link are discussed in Article 2.22.

The optical fibre cable is preferred over other links as :

- They have higher information carrying capacity.
- The optical fibres are made of dielectric materials, therefore transmitter and receiver are electrically isolated.
- The material used is silica glass which is very cheap.
- As the information is transmitted by total internal reflection, the transmission losses are less.
- They are very thin, hence occupy very less space.

2. Medical Applications

The main use of optical fibre in medicine is to illuminate or burn the internal organs of human body and collect the scattered light for formation of image.

In endoscopy a bundle of optical fibre is used to illuminate the internal organs of human body where the sunlight cannot reach. Here light from artificial source of light is guided through the fibre. The light will fall on the organ and will be scattered. This scattered light is collected by another bundle of optical fibres. This scattered light is used for formation of image.

In ophthalmology a laser beam is guided by the fibre is to detach the retina or for vision correction.

A guided laser beam through an optical fibre is also used in angioplasty. A special catheter having three channels – one for guiding laser beam, second for formation of image and third a hollow tube to remove the blocking tissues. The laser beam is used to cut or burn the unwanted tissues.

3. Military Applications

The military equipments, aircrafts, ships, submarines need heavy copper wires for communication equipments. The heavy weight copper wires can be replaced by light weight optical fibres. This reduces the load increasing the overall efficiency of the instruments.

Due to high information carrying capacity they can be used to transmit the video which shows the real situations and helps the ground staff for controlling unmanned vehicles, aeroplanes or missiles.

4. Fibre Optic Sensors

The fibre optic is used to couple the sensors and detectors. The advantages of these sensors are that they are cheap and light weight. These sensors can be used to measure pressure, temperature, stress etc.

- **Temperature Sensor :** The fibre is coated with a thin silicon layer at one end. The silicon layer is backed by a reflective coating as shown in Fig. 2.56. When a light beam is passed through the coating it passes through the silicon layer and is reflected by reflective surface. This reflected light returns to the detector. The absorption of silicon varies with temperature which alters the intensity of the light received by the detector. This variation in the intensity of light is sensed as variation in temperature.

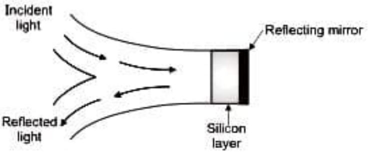


Fig. 2.56 : Temperature sensor

- **Pressure Sensor :** The concept of photoelasticity or induced double refraction is used to design pressure sensor. Fig. 2.57 shows the schematic diagram of pressure sensor.

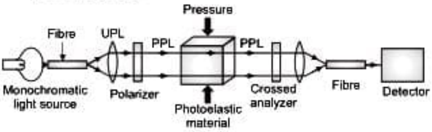


Fig. 2.57 : Pressure sensor

The photoelastic material is kept between crossed polarizer and analyzer. The monochromatic light is guided from source to polarizer using an optical fibre. Then the light is passed through the photoelastic material. The light coming out of the photoelastic material is passed through the crossed (kept at 90° w.r.t. polarizer) analyzer. When no pressure is applied, the axis of polarization remains unaltered and hence no light passes through the analyzer. When mechanical pressure is applied bire fringence is introduced. This gives O-ray and E-ray which are plane polarized in the plane perpendicular to each other. The O-ray will pass through the analyzer. Hence transmission of light occurs which is detected by the detector.

- **Smoke or Pollution Detector :** A smoke detector can be constructed by using optical fibre. A beam of light coming out off a fibre is collected by another fibre kept at some distance. If smoke or dust particles are present between fibres, light will be scattered by smoke particles. This will reduce the amount of light collected by the second fibre. The intensity of light collected will depend on the density of smoke, hence the variation in density will be detected.
- **Interference Sensor :** In this, a single mode fibre is used. A beam splitter is used to divide a laser beam into two parts. Each of these two parts are collected by two separated fibres. One of the fibres acts as sensing fibre and other as reference fibre. The sensing fibre changes the opitcal path of the light travelling through it, due to change in the length or refractive index of the fibre. Fig. 2.58 shows the Mach-Zehnder arrangement of interferroelectric sensor.

  The light entering the fibres is coherent whereas light coming out of the fibres will have phase difference due to change in the optical path due to physical parameter being measured. These two beams interfere to give interference pattern. The measurement of fringe width will give the value of physical parameter being measured.

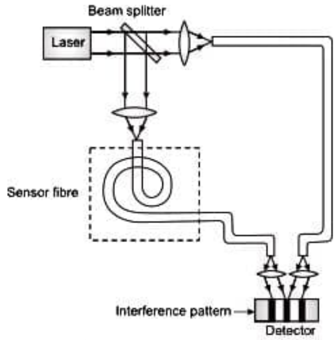


Fig. 2.58 : Interferometric sensor

SUMMARY

- **Interference:** The superposition of two waves of equal amplitude, frequency and a constant phase difference.

  (a) Constructive Interference or Bright Fringe: Whole number multiple of λ or nλ.

  (b) Destructive Interference or Dark Fringe: Odd integer multiple of $\frac{\lambda}{2}$ or $(2n + 1)\frac{\lambda}{2}$.
- **Thin Film:** A film is said to be thin if its thickness is of the order of a few wavelengths.
- In a thin **Film of Uniform Thickness** in reflected system, condition for

  (a) Constructive interference:

  $$2\mu t \cos r = (2n + 1)\frac{\lambda}{2}$$

  (b) Destructive interference:

  $$2\mu t \cos r = n\lambda$$

  (c) In transmitted system, the conditions reverse.
- In a **Wedge-Shaped Film**, the condition for

  (a) Constructive interference:

  $$2\mu t \cos (r + \alpha) = (2n + 1)\frac{\lambda}{2}$$

  (b) Destructive interference:

  $$2\mu t \cos (r + \alpha) = n\lambda$$

  (c) Fringe width:

  $$\beta \approx \frac{\lambda}{2\mu\alpha}$$

  (d) Fringes obtained are: equal in thickness, straight, parallel and equidistant.
- **Newton's Rings**: Fringes of circular in shape with dark fringe at the centre. The width of the fringe decreases with the order of the fringe i.e. as one moves away from the centre.
- **Diameter for**

  (a) Bright ring (reflected light)

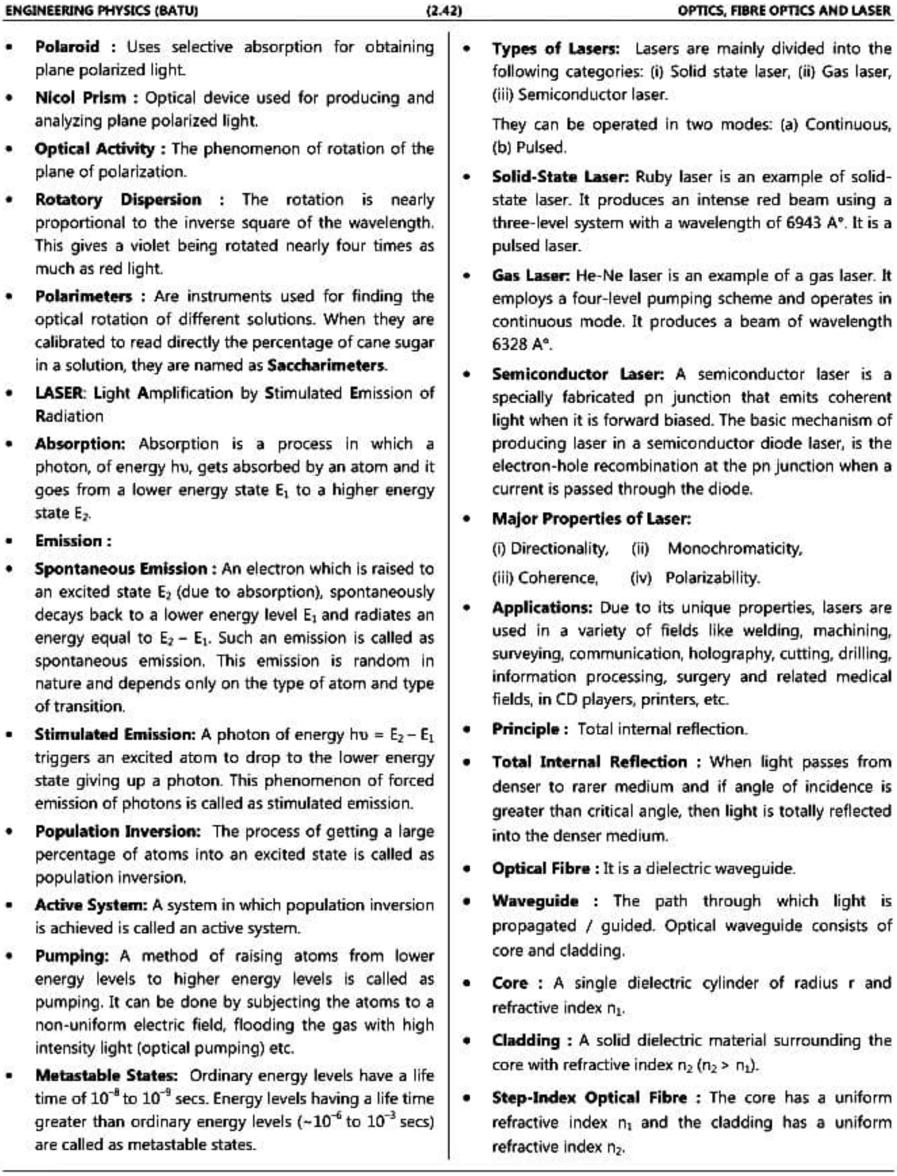
  $$D_n^2 = (2n \pm 1) \cdot 2\lambda R$$

  (b) Dark ring (reflected light)

  $$D_n^2 = 4n\lambda R$$
- **Wavelength of Monochromatic Source of Light**:

  $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$
- **Refractive Index of Liquid:**

  $$\mu = \frac{D'^2_{n+p} - D'^2_n}{D^2_{n+p} - D^2_n}$$
- **Unpolarized Light** has the electric vector vibrating along all possible directions at right angles to the direction of propagation of light.
- **Types of Polarization** (i) Plane, (ii) Circular and (iii) Elliptical
- **Partially Polarized Light:** Mixture of plane polarized light and unpolarized light.
- If the vibrations of the electric vector in a light wave are confined to a single plane, then the light wave is **plane polarized or linearly polarized.**
- **Methods of Production of Plane Polarized Light:** (i) Reflection, (ii) Refraction, (iii) Scattering, (iv) Selective absorption and (v) Double refraction.
- **Brewster's Law:** States that, tangent of the angle of polarization is proportional to the refractive index of the medium i.e. $\mu = \tan i_p$.
- **Double Refraction or Birefringence**: When light passes through anisotropic crystals, it splits up into two rays, O-ray and E-ray.
- **Birefringence** of the crystal is given by, $\Delta\mu = \mu_e - \mu_o$.
- **Positive Crystals** : the velocity of ordinary ray is greater than that of the extraordinary ray ($\mu_e > \mu_o$).
- **Negative Crystals :** the velocity of extraordinary ray is greater than that of the ordinary ray ($\mu_o > \mu_e$).
- **Optic Axis** : O-ray and E-ray travel with the same velocity.



- **Polaroid :** Uses selective absorption for obtaining plane polarized light.
- **Nicol Prism :** Optical device used for producing and analyzing plane polarized light.
- **Optical Activity :** The phenomenon of rotation of the plane of polarization.
- **Rotatory Dispersion :** The rotation is nearly proportional to the inverse square of the wavelength. This gives a violet being rotated nearly four times as much as red light.
- **Polarimeters :** Are instruments used for finding the optical rotation of different solutions. When they are calibrated to read directly the percentage of cane sugar in a solution, they are named as **Saccharimeters.**
- **LASER:** **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation
- **Absorption:** Absorption is a process in which a photon, of energy hυ, gets absorbed by an atom and it goes from a lower energy state E_1 to a higher energy state E_2.
- **Emission :**
- **Spontaneous Emission :** An electron which is raised to an excited state E_2 (due to absorption), spontaneously decays back to a lower energy level E_1 and radiates an energy equal to $E_2 - E_1$. Such an emission is called as spontaneous emission. This emission is random in nature and depends only on the type of atom and type of transition.
- **Stimulated Emission:** A photon of energy $h\upsilon = E_2 - E_1$ triggers an excited atom to drop to the lower energy state giving up a photon. This phenomenon of forced emission of photons is called as stimulated emission.
- **Population Inversion:** The process of getting a large percentage of atoms into an excited state is called as population inversion.
- **Active System:** A system in which population inversion is achieved is called an active system.
- **Pumping:** A method of raising atoms from lower energy levels to higher energy levels is called as pumping. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light (optical pumping) etc.
- **Metastable States:** Ordinary energy levels have a life time of 10^{-8} to 10^{-9} secs. Energy levels having a life time greater than ordinary energy levels ($\sim 10^{-6}$ to 10^{-3} secs) are called as metastable states.
- **Types of Lasers:** Lasers are mainly divided into the following categories: (i) Solid state laser, (ii) Gas laser, (iii) Semiconductor laser.

  They can be operated in two modes: (a) Continuous, (b) Pulsed.
- **Solid-State Laser:** Ruby laser is an example of solid-state laser. It produces an intense red beam using a three-level system with a wavelength of 6943 A°. It is a pulsed laser.
- **Gas Laser:** He-Ne laser is an example of a gas laser. It employs a four-level pumping scheme and operates in continuous mode. It produces a beam of wavelength 6328 A°.
- **Semiconductor Laser:** A semiconductor laser is a specially fabricated pn junction that emits coherent light when it is forward biased. The basic mechanism of producing laser in a semiconductor diode laser, is the electron-hole recombination at the pn junction when a current is passed through the diode.
- **Major Properties of Laser:**

  (i) Directionality, (ii) Monochromaticity,

  (iii) Coherence, (iv) Polarizability.
- **Applications:** Due to its unique properties, lasers are used in a variety of fields like welding, machining, surveying, communication, holography, cutting, drilling, information processing, surgery and related medical fields, in CD players, printers, etc.
- **Principle :** Total internal reflection.
- **Total Internal Reflection :** When light passes from denser to rarer medium and if angle of incidence is greater than critical angle, then light is totally reflected into the denser medium.
- **Optical Fibre :** It is a dielectric waveguide.
- **Waveguide :** The path through which light is propagated / guided. Optical waveguide consists of core and cladding.
- **Core :** A single dielectric cylinder of radius r and refractive index n_1.
- **Cladding :** A solid dielectric material surrounding the core with refractive index n_2 ($n_2 > n_1$).
- **Step-Index Optical Fibre :** The core has a uniform refractive index n_1 and the cladding has a uniform refractive index n_2.

- **Graded-Index Optical Fibre :** The refractive index of the core varies continuously from n_1 at the centre to n_2 at the core-cladding interface.
- **Acceptance Angle :** Light incident within the cone of half angle θ_o.
- **Numerical Aperture :** $\sin \theta_o = \sqrt{n_1^2 - n_2^2}$.
- Modes of propagation :

  (a) Single mode, (b) Multimode.

IMPORTANT FORMULAE

- General condition for constructive interference,

  $x = n\lambda$, where $n = 0, 1, 2 ...$
- General condition for destructive interference,

  $x = (2n + 1)\frac{\lambda}{2}$, where $n = 0, 1, 2 ...$
- For uniform film,
  - Condition for constructive interference,

    $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}, \quad n = 0, 1, 2 ...$
  - Condition for destructive interference,

    $2\mu t \cos r = n\lambda, \quad n = 0, 1, 2 ...$
- For nonuniform (wedge) film,
  - Condition for constructive interference,

    $2\mu t \cos (r + \alpha) = (2n \pm 1)\frac{\lambda}{2}, \quad n = 0, 1, 2 ...$
  - Condition for destructive interference,

    $2\mu t \cos (r + \alpha) = n\lambda, \quad n = 0, 1, 2 ...$
- Fringe width of fringes formed by wedge film,

$$\beta = \frac{\lambda}{2\mu \sin \alpha}$$

- Newton's rings,
  - Diameter of bright fringe,

$$D_n^2 = \sqrt{2\lambda R} \cdot \sqrt{2n \pm 1} \text{ (bright)}$$

  - Diameter of dark fringe, $D_n^2 = 4n\lambda R$
- Wavelength of monochromatic source of light,

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

- Refractive index of liquid, $\mu = \dfrac{D_{n+p}'^2 - D_n'^2}{D_{n+p}^2 - D_n^2}$
- Brewster's Law, $\mu = \tan i_p$
- For polarization by reflection, $i_p + r = \frac{\pi}{2}$
- The transmitted light through polarizer (Law of Malus),

$$I = I_o \cos^2 \theta$$

- Refractive index of O-ray, $\mu_o = \dfrac{\sin i}{\sin r_o} = \dfrac{c}{v_o}$
- Refractive index of E-ray, $\mu_e = \dfrac{\sin i}{\sin r_e} = \dfrac{c}{v_e}$
- Specific rotation

$$s = \frac{10\theta}{l \times c}$$

- Rate of absorption, $R_{12} = P_a N_1$.
- Rate of stimulated emission, $R_{21} = P_e N_2$.
- At equilibrium, $P_a = P_e$.
- Population inversion, $N_2 > N_1$.
- Frequency of photons emitted, $\upsilon = \dfrac{E_2 - E_1}{h}$.

UNSOLVED PROBLEMS

1. A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.

   [Ans. 3.926×10^{-5} cm]
2. Two pin holes separated by a distance of 0.5 mm are illuminated by a monochromatic light of wavelength 6000 A°. An interference pattern is obtained on a screen placed at a distance of 100 cm from the pin holes. Find the distance on the screen between the fifth and tenth dark fringes. **[Ans.** 0.6 cm]
3. An oil drop of volume 0.2 cc is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain first dark band whose centre has wavelength of 5.5×10^{-5} cm. Find the refractive index of oil. **[Ans.** 1.375]
4. A soap film of refractive index $\frac{4}{3}$ and of thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference of the dark band.

   [Ans. n = 6]

5. The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used. **[Ans.** Dark, 6015 A°]

6. In a Newton's rings experiment, the diameter of the 5th ring is 0.336 cm and the diameter of the 15th ring is 0.590 cm. Find the radius of curvature of the plano convex lens, if the wavelength of light used is 5890 A°. **[Ans.** 99.82 cm]

7. In a Newton's rings experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890 A°, the diameter of the third dark ring is 0.32 cm. The light is incident normally. **[Ans.** 144.9 cm]

8. In Newton's rings, the diameter of a certain bright ring is 0.65 cm and that of tenth ring beyond it is 0.95 cm. If λ = 6000 A°, calculate the radius of curvature of a convex lens surface in contact with the glass plate. **[Ans.** 200 cm]

9. In a Newton's rings experiment, a drop of water $\left(\mu = \frac{4}{3}\right)$ is placed between the lens and the plate. In that case, the diameter of the 10th ring was found to be 0.6 cm. Calculate the radius of curvature of the face of the lens in contact with the plate, given λ = 6000 A°. **[Ans.** 200 cm]

10. Newton's rings are observed in reflected light of λ = 5900 A°. The diameter of the 5th dark ring is 0.4 cm. Find the radius of curvature of the lens and the thickness of the air film. **[Ans.** 35.59 cms, 0.000295 cm]

11. In a Newton's ring experiment, the diameters of 4th and 12th dark rings are 0.4 cm and 0.7 cm respectively. Calculate the diameter of 20th dark ring. **[Ans.** 0.894 cm]

12. In a Newton's rings experiment, the source emits two wavelengths λ_1 = 6000 A° and λ_2 = 4500 A°. It is found that nth dark ring due to λ_1 coincides with (n + 1)th dark ring due to λ_2. If the radius of curvature of the curved surface is 90 cm, find the diameter of nth dark ring for λ_1. **[Ans.** 0.2538 cm]

13. If the diameter of nth dark ring in a Newton's ring experiment changes from 0.3 cm to 0.25 cm, as a liquid is placed between the lens and the plate, calculate the value of μ of the liquid. **[Ans.** μ = 1.44]

14. A wedge-shaped air film, having an angle of 45 seconds, is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between the consecutive fringes is 0.12 cm, calculate the wavelength of light used. **[Ans.** 5233 A°]

15. Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds, of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm. **[Ans.** 89.1 seconds]

16. Two rectangular pieces of a plane glass are laid one upon the other and a thin wire is placed between them, so that a thin wedge shaped air film is formed between them. The plates are illuminated with sodium light of λ = 5893 A° at normal incidence. Bright and dark bands are formed, there being 10 of each per cm length of the wedge measured normal to the edge in contact. Find the angle of the wedge. **[Ans.** 2.94×10^{-4} radians]

17. Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes. **[Ans.** 0.295 cm]

18. Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°. **[Ans.** 1.03×10^{-4} cm]

19. If the plane of vibrations of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light. $\left[\textbf{Ans. } \frac{I_e}{I_o} = 3\right]$

20. A beam of light travelling in water strikes a glass plate which is also immersed in water. When the angle of incidence is 51°, the reflected beam is found to be polarized. Calculate the refractive index of glass. **[Ans.** 1.235]

21. A glass plate is used as a polarizer. Find the angle of polarization for it. Also find the angle of refraction, given μ for glass = 1.54. **[Ans.** 57°, 33°]

22. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is maximum. Through what angle must either sheet be turned so that the intensity becomes one half the initial value ? **[Ans.** 45°, 135°**]**
23. The refractive index for plastic is 1.25. Calculate the angle of refraction for a ray of light inclined at polarizing angle. **[Ans.** 38.6°**]**
24. A beam of light is passed through two Nicol prisms in series. In a particular setting, maximum light is passed by the system and it is 500 units. If one of the Nicols is now rotated by 20°, calculate the intensity of transmitted light. **[Ans.** 441.5 units**]**
25. Two Nicol prisms are oriented with their principal planes making an angle of 30°. What percentage of incident unpolarized light will pass through the system ? **[Ans.** 37.5 %**]**
26. A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through (i) 45°, (ii) 90° ? **[Ans.** 0.5, 0**]**

EXERCISE

1. Explain the phenomena of interference.
2. What is constructive and destructive interference ?
3. Derive the conditions for constructive and destructive interference.
4. Explain the phenomenon of interference in thin films in reflected light.
5. What are Newton's rings ? Explain how they are formed.
6. Explain the formation of colours in thin films.
7. Explain the phenomenon of interference in thin film in transmitted light.
8. How can Newton's rings be obtained in the laboratory ? How will you use them to measure the wavelength of sodium light ?
9. Explain the theory and the experimental arrangement of Newton's rings experiment.
10. What have you understood by non-reflecting films ? Explain.
11. In Newton's rings, show that the radii of dark rings are proportional to the square root of natural numbers.
12. When seen by reflected light, why does an excessively thin film appear to be perfectly black when illuminated by a white light ?
13. Explain, why colours are not observed in the case of a thick film when illuminated by a white light.
14. How can Newton's rings be used to determine the refractive index of a liquid ? Derive the formula used.
15. Prove that in reflected light Newton's rings, the diameters of bright rings are proportional to the square root of the odd natural numbers.
16. How can Newton's rings be obtained in the laboratory ? Prove that for Newton's rings in reflected light, the diameters of dark rings are proportional to the square root of natural numbers.
17. Explain the term polarization of light.
18. Define plane of polarization and plane of vibration. Explain a method to show that light waves are transverse.
19. Distinguish between polarized and unpolarized light.
20. State Brewster's law and use it to prove that when light is incident on a transparent substance at the polarizing angle, the reflected and refracted rays are at right angles to each other.
21. Explain how you would obtain plane polarized light by reflection.
22. What is pile of plates ? Explain how it can be used for producing plane polarized light.
23. What is polarizing angle ? Explain.
24. Explain the phenomenon of double refraction in calcite.
25. Describe the construction and working of a Nicol prism.
26. What is a Nicol prism ? Explain how a Nicol prism can be used as an analyzer and polarizer.
27. Explain giving diagrams the nature of refraction observed in the case of calcite crystal when:
    (a) Optic axis is parallel to the refractive surface and lying in the plane of incidence (normal incidence).
    (b) Optic axis is perpendicular to the refracting surface and lying in the plane of incidence (normal incidence).
28. Give Huygen's construction for ordinary and extraordinary wavefronts when the beam of light is refracted through a doubly refracting crystal when the optic axis is inclined to the crystal surface and lying in the plane of incidence (normal incidence).
29. What does the numerical aperture indicate ?
30. Give advantages of using optical fibre as compared to conventional cable for telecommunication.
31. What is optical activity? Explain, how Laurentz's half shade polarimeter can be used for measuring specific rotation?
32. Explain the operation of Ruby laser with a neat labelled diagram.
33. Explain the following terms:
    (i) Spontaneous emission (ii) Stimulated emission
    (iii) Population inversion.

34. Explain action of gas laser. How does stimulated emission take place with exchange of energy between Helium and Neon atoms ?
35. What is population inversion ? Explain the operation of He - Ne laser.
36. What are the different uses to which laser beams are put in industry, medicine ?
37. Define and explain the terms:
    (i) Pumping (ii) Active systems.
38. Write a note on use of lasers in fibre communication systems and information technology.
39. Explain the elements of optical fibre communication link.
40. What are various parts in the optical fibre communication system ?
41. Explain total internal reflection and its relation with the working of optical fibre.
42. Give constructional features of optical fibre.
43. Describe various parts of optical fibre ?
44. Give name of different types of optical fibre and their structure.
45. What do you mean by
    (i) Monomode fibres, (ii) Multimode fibres.
46. Explain 'Numerical Aperture' and arrive at the expression for numerical aperture.

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Animation of double refraction:

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UNIVERSITY QUESTIONS

December 2017

1. Derive an expression for the optical path difference for the reflected rays in a thin film of constant thickness and hence find the conditions for maxima and minima. **[6]**
2. What is double refraction? Explain the difference between ordinary ray (O-ray). And extra ordinary ray (e-ray) **[6]**
3. What is population inversion and stimulated emission ? Calculate the acceptance angle of an optical fibre where the refractive index of core is 1.55 and that of cladding is 1.50. **[4+2]**

May 2018

1. In case of Newton's rings in reflected light show that diameter of bright rings is proportional to the square root of odd natural numbers. In Newton's rings, the diameter of a certain bright ring is 0.65 cm and that of tenth ring is 0.95 cm. If λ = 6000 A°, calculate the radius of curvature of a convex lense.
2. Give the diagrammatic representation of polarized and unpolarized light. **[6]**
   Explain the method of producing plane polarized light by reflection.
3. Explain the construction and working of He-Ne laser with neat diagram. **[6]**

December 2018

1. In case of Newton's rings, prove $D_n \alpha \sqrt{n}$, where D_n is diameter of nth dark ring. **[6]**
2. Explain Double refraction using Huygen's wave theory of light. **[6]**
3. Explain the construction and working of Ruby laser with neat diagram. **[6]**

May 2019

1. Prove that for Newton's rings in reflected light, the diameter of dark ring is proportional to the square root of natural numbers. **[6]**
2. Explain construction and working of Ruby laser with neat diagram. **[6]**
3. Obtain mathematical expression for acceptance angle and numerical aperture. **[6]**

✠ ✠ ✠

UNIT III

ELECTRON OPTICS, NUCLEAR PHYSICS AND QUANTUM MECHANICS

3.1 INTRODUCTION TO ELECTRON OPTICS

- It is a known fact that **Cathode Rays** consist of electrons moving with a high speed. Electrons enter into the constitution of any kind of matter. Therefore, before commencing the study of any electronic device, it is imperative to understand the behaviour or motion of the electron under the action of electric and magnetic fields. The first part of the chapter is devoted to this.
- The properties of the electrons of being deflected by electric and magnetic fields and of producing scintillations on a fluorescent screen are made use of in the construction and action of a CRO and an electron microscope.
- The electron microscope has gained a place as an invaluable device to professionals dealing with the ultra small in a number of spheres. The second part of the chapter deals with these instruments and the principles of focusing of electrons required for the functioning of an electron microscope.
- **Microscopy** is now an invaluable tool for the study of the finer and smaller details of matter. A preliminary discussion of scanning electron microscopy and scanning tunneling microscopy is given here.
- The last part of the unit concentrates on positive rays and their analysis, which led to the discovery of isotopes with the help of mass spectroscopy. Details of the most elegant of the mass spectrographs, the Bainbridge mass spectrograph are given in this section.

3.2 MEASUREMENT OF 'e/m' BY THOMSON'S METHOD [Dec. 18]

In 1897, J. J. Thomson succeeded in determining the e/m ratio of electrons.

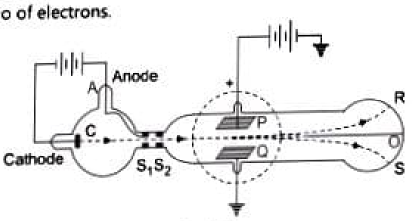


Fig. 3.1

1. **Description and Working :**

- Cathode rays are produced in the discharge tube, when a high potential difference is applied between the cathode C and the anode A.
- The rays then pass through two slits or metal diaphragms S_1 and S_2 maintained at anode potential. The purpose of the slits is to obtain a fine beam of electrons.
- The beam strikes the surface of the discharge tube normally at O. As it is coated with a fluorescent material, fluorescence is produced.
- P and Q are two plates between which electric field is produced by applying a suitable potential difference. This field is perpendicular to the plates and is directed from the positive plate to the negative.
- Due to the electric field, the electronic beam is deflected upwards (P being positive). Now, the fluorescent spot is obtained at R.
- By placing the tube between the pole pieces of a powerful electromagnet, a magnetic field is applied in a direction perpendicular to the plane of the plates and the direction of motion of electrons.
- The direction of the field is adjusted so that the electron beam is deflected downwards. The fluorescent spot is now obtained at S.
- The direction of deflection of electrons under the influence of a magnetic field is obtained by Fleming's left hand rule.

2. **Theory :**

- **Deflection of Electron Beam by Magnetic Field :**
- The electric and magnetic fields are firstly switched off. The position of the spot O on the screen for the undeflected beam is noted. The magnetic field is now applied in a direction perpendicular to the direction of motion of electrons.
- If 'B' is the intensity of the magnetic field, 'e' the charge of the electron, 'v' the velocity of the electron, then BeV is the magnetic force acting on the electron. This force is directed perpendicular to the motion of the electrons and magnetic field. (Fleming's law). Under the effect of

a constant uniform magnetic force, the electrons take up a circular path.

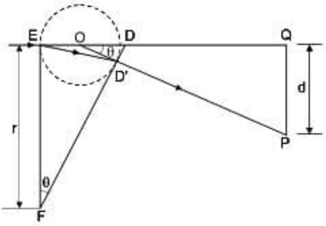


Fig. 3.2

- Beyond the region of influence of the magnetic field (dotted region), the electron beam emerges out in a straight line, tangential to the arc at the point of emergence as shown in Fig. 3.2.
- If r is the radius of the circular path, then the centripetal force acting on the electron beam is $\frac{mv^2}{r}$, where m is the mass of the electron and B the strength of the magnetic field. Under the influence of the magnetic field, the magnetic force supplies the centripetal force.

Hence, $$B e v = \frac{mv^2}{r}$$

or $$\frac{e}{m} = \frac{v}{Br} \qquad \text{... (3.1)}$$

- Thus, if r and v are known then the specific charge ratio (e/m) can be determined.
- **Calculation of r :** From Δ EDF, ∠ EDF = 90° – θ

As OQ is tangent to FD, hence ∠ OD'D = 90°

Thus from Δ ODD',

$$\angle DOD' = \theta$$

From Δ POQ, $$\tan\theta = \frac{PQ}{OQ} = \frac{d}{OQ} \qquad \text{... (i)}$$

From Δ EDF, $$\tan\theta = \frac{\text{arc } ED'}{r} \approx \frac{ED}{r} \qquad \text{... (ii)}$$

Hence, from (i) and (ii), we get

$$\frac{d}{OQ} = \frac{ED}{r}$$

or $$r = \frac{ED \times OQ}{d} \qquad \text{... (3.2)}$$

ED is the region of influence of the field. This is taken equal to the length of the plate P or Q in Fig. 3.1.

The value of OQ is given on the discharge tube, r can therefore be calculated from relation (3.2).

- **Determination of v :** Under the influence of crossed electric and magnetic fields, the beam strikes the screen at the same spot O as that of the undeflected beam. In such a case, the force on the electron due to the electric field (Ee) is balanced by the deflecting force (Bev) due to the magnetic field.

Hence, $$B e v = E e$$

i.e., $$v = \frac{E}{B} \qquad \text{... (3.3)}$$

Substituting values of v and r in (3.1), we have,

$$\frac{e}{m} = \frac{E}{B^2} \cdot \frac{d}{(ED \times OQ)} \qquad \text{... (3.4)}$$

Substituting values of E, d, B, ED and OQ, e/m can be calculated. Value of e/m of an electron is 1.7589×10^{11} coulomb/kg.

3. Results :

- Value of e/m is found to be a constant, independent of the material of the cathode and the nature of the gas in the tube. So, nature of particles in the cathode rays are of the same kind irrespective of their origin and velocity acquired.
- Comparing the value of e/m of the electron with that of e/m of the hydrogen ion, it is seen that the first is 1840 times that of the latter. This implies that either charge of electron is 1840 times charge of hydrogen ion, mass being same or mass of electron is $\frac{1}{1840}$ of mass of hydrogen ion, charge being the same. To ensure this, e is measured separately.

3.3 DETERMINATION OF ELECTRONIC CHARGE BY MILLIKAN'S OIL DROP METHOD

- The set up was developed by American scientist R. A. Millikan in 1917 to measure charge of an electron. The Fig. 3.3 shows the set-up of the instrument.

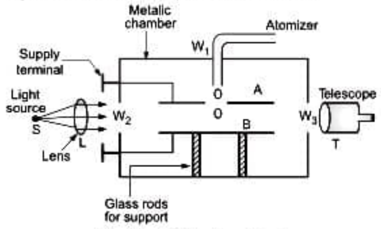


Fig. 3.3 : Millikan's method

- **Set-up**
  - A and B are plane metallic disc of about 20 cm in diameter placed at a distance pf 1.6 cm. The discs are clamped by insulating rods of glass or ebonite so that they remain perfectly parallel to each other.
  - The discs are placed in metallic chamber provided with three windows W_1, W_2 and W_3. A variable voltage is applied between the plates A and B. The upper plate A is provided with a small hole through which small droplets of a heavy non-violate oil can be introduced between two plates through a spring atomizer.
  - The droplets come slowly which gets charged due to friction in the spray process. The space between two plates is illuminated by high intensity light beam. This illuminates the oil droplets which can be seen by a telescope which is connected to a scale for measurement.
  - When illuminated the oil drops appear as brilliant spots on a dark background.

Procedure :

- The experiment is performed in two stages. In first stage the experiment is performed in the absence of electric field between the plates. When oil is dropped, the oil drop moves down under the influence of gravitational force. Due to air friction, soon electron will reach terminal velocity. i.e. its velocity will not increase further.
- At this stage the net downward gravitational force equals the force offered by air resistance. If 'ρ' is the density of oil, 'g' is acceleration due to gravity and 'a' the radius of the oil drop then,

  Weight, $$W = \left(\frac{4}{3}\pi a^3\right)\rho\, h \qquad \ldots (3.5)$$

- By Archimede's principle the upward thrust experience by oil drop due to displaced air is,

  $$T = \left(\frac{4}{3}\pi a^3\right)\sigma\, g \qquad \ldots (3.6)$$

  where, 'σ' is density of air.

- If a spherical body of radius a falls under gravity in fluid having a coefficient of viscosity 'η', then by Stroke's law the resistive force due to medium

  $$f = \sigma \pi \eta\, a\, v_1 \qquad \ldots (3.7)$$

  where, v_1 is the velocity of the drop.

- When the equilibrium is reached and the velocity of the drop becomes uniform the resultant force on the drop is zero, that is.

  $$W = T + f$$

  i.e. $$\frac{4}{3}\pi a^3 (\rho - \sigma)\, g = \sigma \pi \eta\, a\, v_1 \qquad \ldots (3.8)$$

- In the second part of the experiment, an electric field E is applied between the plates. Now an additional upward electric force qE acts on the oil drop. Due to which the terminal velocity reduces to v_1. Therefore, the equation of motion for drop becomes.

  $$\frac{4}{3}\pi a^3 (\rho - \sigma)\, g - qE = \sigma \pi \eta\, a\, v_2 \qquad \ldots (3.9)$$

  where, 'q' is charge on oil drop.

  From equation (3.8) and (3.9),

  $$qE = \sigma \pi \eta\, a\, (v_1 - v_2)$$

  or $$q = \frac{\sigma \pi \eta\, a}{E}(v_1 - v_2) \qquad \ldots (3.10)$$

  Again from equation (3.8),

  $$a^2 = \frac{g\, \eta\, v_1}{2\, g\, (\rho - \sigma)}$$

  or $$a = \left[\frac{g\, \eta\, v_1}{2\, g\, (\rho - \sigma)}\right]^{1/2} \qquad \ldots (3.11)$$

  Substituting value of a in equation (3.10),

  $$q = \frac{\sigma \pi \eta}{E}\left[\frac{g\, \eta\, v_1}{2\, g\, (\rho - \sigma)}\right]^{1/2} (v_1 - v_2) \qquad \ldots (3.12)$$

- The value of charge of an electron as worked out by Millikan's is 1.59×10^{-19} C., which agrees with recent experimental values.

3.4 MASS SPECTROGRAPH

- Isotopes are elements having the same atomic number but different atomic weights.

  For Example : ${}_1H^1$ and ${}_1D^2$; ${}_{92}U^{233}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$; ${}_{10}Ne^{20}$ and ${}_{10}Ne^{22}$

- As the atomic number of isotopes is the same, they have the same electronic configuration and hence the same chemical properties. They can therefore be separated by physical methods and not by chemical methods.
- To detect the presence of isotopes, to find an accurate value of isotopic masses and their abundance, F.W. Aston, an English Physicist, devised a mass spectrograph. This spectrograph brought about a

separation between the isotopes on the basis of their masses. This was followed by Dempster's which has recently been superseded by Bainbridge's magnetic deflection mass spectrograph.

3.5 BAINBRIDGE MASS SPECTROGRAPH

[Dec. 17, May 19]

- Bainbridge used a power electromagnet and a velocity selector in his spectrograph and was able to obtain, a high resolving power, precise symmetric images and a linear mass scale which could not be obtained in Aston's or Dempster's spectrographs.

Principle :

- Whatever be the velocities of the ions in the process of their generation, they are made perfectly homogeneous in velocity by the use of a special device called the **Velocity Selector**. They are then subjected to an extensive, transverse magnetic field and are brought to focus on a photographic plate.

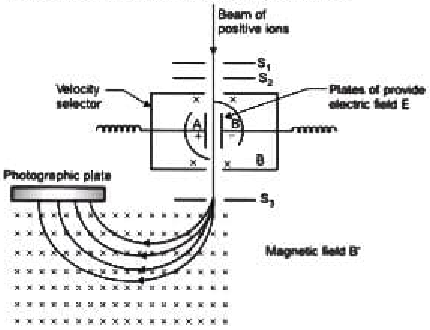


Fig. 3.4 : Bainbridge spectrograph

Apparatus :

- The given beam of ions is collimated by two narrow parallel slits S_1 and S_2. It is then passed through a velocity selector, which consists of a transverse electric field E which is produced by maintaining plates A and B at a suitable p.d.
- Simultaneously a magnetic field B is applied perpendicular to both E and the motion of the ions. The magnetic field is obtained by an electromagnet represented by the dotted circle.
- The velocity selector allows only those ions to pass undeviated which possess the same velocity 'v' given by the following relation.
- Inside the selector,

$$\text{Electric force} = \text{Magnetic force}$$

$$E q = B q v$$

(q is the charge of the ion moving with a velocity v).

$$\therefore \quad v = \frac{E}{B} \qquad \text{... (3.13)}$$

- All other ions bend away the straight path due to the unbalanced effect of one of the two opposing fields.
- The ions emerging from slit S_3, at the exit of the selector, are introduced into a uniform magnetic field of intensity B' acting at right angles to the plane of the paper.
- Under the influence of this field they travel along circular paths which are governed by the following relation.

Magnetic force = Centripetal force

$$\text{i.e.,} \quad B' q v = \frac{m v^2}{r} \qquad \text{... (3.14)}$$

m is the mass of the ion whose circular path has a radius r.

From (3.13) and (3.14), we get

$$B' q = \frac{m}{r} \frac{E}{B}$$

$$\frac{q}{m} = \frac{E}{B \cdot B'} \cdot \frac{1}{r} \qquad \text{... (3.15)}$$

- As E, B, B' are constants, q/m (specific charge ratio) is directly proportional to 1/r or $m \propto r$ and the mass scale is linear. Hence, after describing semi-circles, if the ions are made to fall on a photographic plate, they will strike it at different points depending on the value of mass. Lighter particles will trace small semi-circles while heavier ones will trace larger semi-circles. Traces are obtained on the photographic plate with the mass scale being linear.
- Presence of isotopes is therefore detected by the production of spots on the photographic plate. Mass numbers of the isotopes can be found by comparing the plate with a standard calibrated plate. Relative abundance of the isotopes in a given beam of ions can be found by studying the relative intensity of the spots on the photographic plate.

Hence a Bainbridge mass spectrograph is used :

- To detect the presence of isotopes in a given beam of positive ions,
- To determine the mass number of the isotopes,
- To find out the relative abundance of the isotopes in the given beam of positive ions.

SOLVED PROBLEMS

Problem 3.1 : *In a Bainbridge mass spectrometer, if the magnetic field in the velocity selector is 1 wb/m² and ions having a velocity of 0.4 × 10⁷ m/sec pass undeflected, find the electric field in the velocity selector.*

Data : $v = 0.4 \times 10^7$ m/sec., $B = 1$ wb/m²

Formula : $E = v \cdot B$

Solution : $E = 0.4 \times 10^7 \times 1 = \boxed{4 \times 10^6 \text{ V/m}}$

Problem 3.2 : *Singly ionised magnesium atoms enter a Bainbridge mass spectrograph with a velocity of 3 × 10⁵ m/sec. Calculate the radii of the paths followed by the three most abundant isotopes of masses 24, 25, 26 when the magnetic flux density is 0.5 wb/m².*

Data : $v = 3 \times 10^5$ m/sec., $B = 0.5$ wb/m², $q = 1.6 \times 10^{-19}$

Formula : $R = \frac{mv}{Bq}$

Solution : Mass of single ionised atom of Mg i.e.

$$m_{24} = \frac{24}{6.02 \times 10^{26}} \text{ kg} = 3.987 \times 10^{-26} \text{ kg}$$

$$R_{24} = \frac{M_{24} \cdot v}{B \cdot q} = \frac{3.987 \times 10^{-26} \times 3 \times 10^5}{0.5 \times 1.6 \times 10^{-19}}$$

$$= 14.95 \times 10^{-2} = 0.1495 \text{ m}$$

As $R \propto m$

$\therefore \quad \frac{R_{24}}{R_{25}} = \frac{m_{24}}{m_{25}}$

i.e., $R_{25} = \frac{m_{25}}{m_{24}} \cdot R_{24}$

$$= \frac{25}{24} \times 0.1495 = \boxed{0.1557 \text{ m}}$$

Similarly, $R_{26} = \frac{26}{24} \times 0.1495 = \boxed{0.1619 \text{ m}}$

Problem 3.3 : *A mixture of neon isotopes (Ne²⁰ and Ne²¹) is analysed using a Bainbridge mass spectrometer. Calculate the linear separation of isotopes when the field acting on the velocity selector is 80 kV/meter and the magnetic flux density is 0.55 weber/m².*

Data : $E = 80$ kV/meter; $B = 0.55$ weber/m²

Formulae : $v = \frac{E}{B}$, $R = \frac{mv}{Bq}$

Solution : $R = \frac{m\,(E/B)}{B\,q} = \frac{m\,E}{q\,B^2}$

$$m_{20} = \frac{20}{6.025 \times 10^{26}} \text{ kg}$$

$$= 3.3195 \times 10^{-26} \text{ kg}$$

$$R_{20} = \frac{3.3195 \times 10^{-26}}{1.6 \times 10^{-19}} \times \frac{80 \times 10^3}{(0.55)^2}$$

$$= 548.678 \times 10^{-4}$$

$$= 0.0549 \text{ m}$$

As $R \propto m$

$\frac{R_{21}}{R_{20}} = \frac{m_{21}}{m_{20}}$

$\therefore \quad R_{21} = \frac{m_{21}}{m_{20}} \cdot R_{20}$

$\therefore \quad R_{21} - R_{20} = \frac{m_{21}}{m_{20}} \cdot R_{20} - R_{20}$

$$= \left(\frac{m_{21}}{m_{20}} - 1\right) R_{20}$$

$$= \left(\frac{21}{20} - 1\right) \times 0.0549 = \frac{0.0549}{20}$$

$$R_{21} - R_{20} = 0.00275 \text{ m}$$

The separation on the photographic plate is double that of the radii difference.

∴ Linear separation of isotopes on the photographic plate

$$= 2 \times 0.00275 = \boxed{0.0055 \text{ m}}$$

3.6 INTRODUCTION TO NUCLEAR PHYSICS

- Rutherford, from his experiment on scattering of α-particles, suggested that an atom consists of a central nucleus surrounded by extra-nuclear electrons.
- The nucleus is positively charged and this charge is due to the protons present in the nucleus. The number of protons in the nucleus of an atom gives the atomic number 'Z' of the element to which the atom belongs.
- The nucleus also contains neutrons, which are electrically neutral. The number of neutrons in the nucleus is given by A – Z, where, 'A' is the mass number of the element.
- Nuclear size is about 10^{-15} m and the atomic size is about 10^{-11} m.
- An atom is electrically neutral, so the number of extra-nuclear electrons in an atom is always equal to the number of protons in the nucleus.
- A nucleus of an element is characterized by the mass number 'A' and the atomic number 'Z' of the element.
- Nuclei of mass number upto 25 are called light nuclei. Nuclei of mass number between 25 and 85 are called intermediate nuclei and those of mass number above 85 are called as heavy nuclei.

- For elements of low mass number, the number of protons is nearly equal to the number of neutrons in the nucleus. e.g. for sodium, Z = 11, A = 23. So number of protons in sodium nucleus is 11 and number of neutrons is 12.
- Hence the neutron-proton ratio (n : p) for sodium is nearly equal to unity. Such nuclei, for which n : p ratio is nearly unity, are called stable nuclei. Light nuclei are stable. But with increase of mass number, the number of neutrons exceeds that of protons and n : p ratio exceeds unity. Such nuclei, for which n : p ratio exceeds 1.5, are called unstable nuclei.
- Heavy nuclei are unstable. e.g. for ${}_{92}U^{238}$, A = 238, Z = 92. Hence, n : p ratio is more than 1.5. So uranium is unstable. It exhibits radioactivity and disintegrates till stable end products are formed.
- Henry Becquerel discovered that uranium gave out some type of radiations that could affect a photographic plate wrapped in a thick black paper. It was found that these radiations are highly penetrating, they ionize gases and cause scintillations on a fluorescent screen.
- The substances which emit these radiations are said to be radioactive. e.g. uranium, radium, polonium, radon, etc. The phenomenon of spontaneous emission of radiation from a substance is called as **Radioactivity**. All naturally occurring elements with atomic numbers greater than 82 are found to be radioactive because their $\frac{n}{p}$ ratio exceeds 1.5.
- The nuclear mass is the weight of the nucleus and it is equal to the sum of the masses of the neutrons and protons present in the nucleus.
- As the size of the nucleus is extremely small and its mass is very large, the nuclear density is enormously high, about 10^{17} kg/m^3.
- As mass of electron is negligible, the whole mass of an atom can be taken to be concentrated in its nucleus.
- The ordinary chemical and physical properties of elements are to be attributed to peripheral electrons in their atoms. An atom can be singly ionized by removing one electron and it will then have one excess positive charge.
- When all the electrons of an atom are removed, the bare nucleus will be left behind with only the positive charge, and even then the atom still retains its individuality and intrinsic nature.
- When the nucleus itself is tampered with and its constituent particles are altered in kind and manner, the original atom ceases to exist giving birth to a new one. This phenomenon of conversion of one element into another is known as disintegration or transmutation of elements.
- When transmutation is provoked by artificial means, it is called as artificial transmutation. It seemed possible that if atoms were bombarded with energetic particles, one of the latter might penetrate into a nucleus and cause transmutation.
- Alpha particles from natural radio nuclides were found to be effective for causing transmutation because of their relatively large energy and momentum. To reduce the probability of scattering of bombarding alpha particles and to increase the probability of disintegration, lighter elements were used as targets.
- The first artificial transmutation reaction observed by Rutherford was, when nitrogen was bombarded with α-particles. This transmutation can be represented as,

$${}_{7}N^{14} + {}_{2}He^{4} \rightarrow [{}_{9}F^{18}] \rightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

Some of the artificially transmuted elements were found to be radioactive.

e.g. $${}_{13}Al^{27} + {}_{2}He^{4} \rightarrow [{}_{15}P^{31}] \rightarrow {}_{15}\overset{\bullet}{P}^{30} + {}_{0}n^{1}$$

${}_{15}\overset{\bullet}{P}^{30}$ is radioactive and it disintegrates as

$${}_{15}\overset{\bullet}{P}^{30} \rightarrow {}_{14}Si^{30} + {}_{1}e^{0} + \upsilon \text{ (neutrino)}$$

(positron)

Such reactions led to the discovery of artificial radioactivity.

3.7 PARTICLE DETECTOR – G.M. COUNTER

- A radioactive material keeps on radiating particles like α, β and γ, which cannot be sensed by humans directly. But their presence affects humans directly as well as indirectly, so their measurement becomes essential.
- For detecting and measuring intensity of these radiations some indirect method must be employed.
- When these radiations are passed through gases they have ability to ionise them. This property of the radiations can be employed to detect them.
- The commonly used radiation detectors are ionisation chamber, proportion counter, Geiger-Muller (G.M.) counter, cloud chamber etc.
- A G.M. counter uses a glass tube called G.M. tube along with electrical circuits, needed to amplify the current and display it. The tube consists of a rugged metal case enclosed in the glass tube.

- The hollow metal case acts as cathode. A fine metal wire passing through the centre of the tube acts as anode. The tube is evacuated and then filled with mixture of Argon (90%) at 10 cm pressure and ethyl alcohol vapour (10%) at 1 cm pressure. One end of the tube is enclosed with a thin sheet of mica which serves as window for radiations.
- A d.c. potential difference of about 1200 V is applied between metal case (cathode) and the wire (anode). The voltage is adjusted below the break down voltage of the gas. Fig. 3.5 shows schematic arrangement of G-M counter.

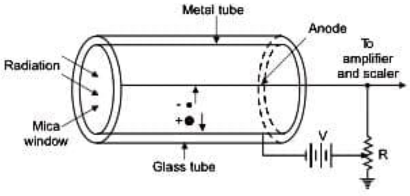


Fig. 3.5 : G-M counter

- When a high energy particle enters the G-M tube it inoises one or more argon atoms. The electron will be attracted by anode wire whereas positive ions will be attracted by negative of the supply. These moving charges further ionises the argon atoms.
- As a result of this a current pulse will pass. This current pulse through resistance R produces a voltage pulse of the order of 10 μV. An electronic pulse amplifier amplifies this weak voltage pulse to a voltage value between 5 to 50 V.
- These amplified pulses are applied to a counter. As each incoming radiation produces a pulse of current, the number of incoming radiations can be counted.
- The Fig. 3.6 shows a graph of counts per minute as a function of voltage for voltages less than 1000 V. There is no discharge and hence will show zero count. Between 1000 V to 1200 V the number of pulses are proportional to the voltage.
- Above 1200 V the number of counts remain constant for certain range of voltage. This is known as **Plateau Region**. The plateau region is used for normal operation of G-M counter.

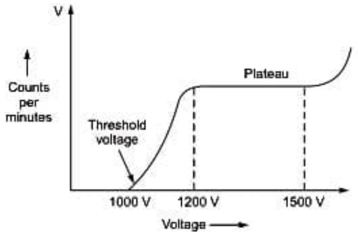
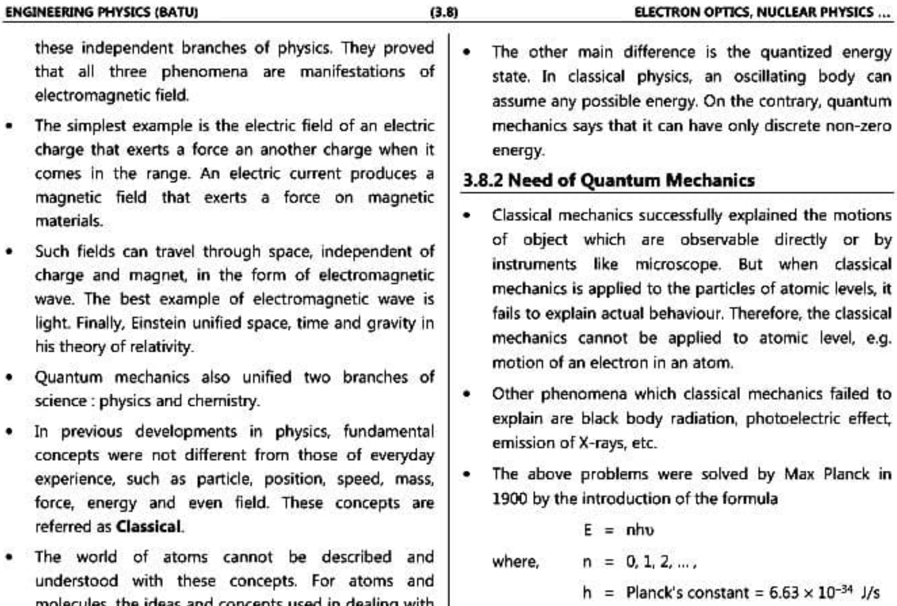


Fig. 3.6 : Characteristics of G-M counter

- Ethyl alcohol vapour is used to prevent undesirable avalanching. A G-M counter can count upto 500 particles per second.
- The main drawback of G-M counter is the **Dead Time**, the time taken by the tube to recover between counts.
- The **Dead Time** of G-M counter is about 200 μs. If the radiation density is high, the tube will not have time to recover and hence some of the particles may not be counted.
- The G-M counter can be used to count β and γ-radiations and α-particles with some modifications in the tube.

3.8 INTRODUCTION TO QUANTUM MECHANICS

- The most outstanding development in modern science is the conception of quantum mechanics. The quantum mechanics is better than Newtonian classical mechanics in explaining the fundamental physics. There was big development in physics between the time of Newton and the time of quantum mechanics.
- Newton showed that the motion of planets and the free fall of an object on earth is governed by the same law. Thus, he unified terrestrial and celestial mechanics. This was in contrast to ancient belief that the world of the earth and heaven is governed by different laws.
- It was earlier believed that the heat is some peculiar substance called **Caloric**, which flows from a hot object to a cold object. But latter it was proved that the heat is the random motion or vibration of constituents of matter. Thus, thermodynamics and mechanics were unified.
- For a long time, the phenomena of electricity, magnetism and light were treated as independent branches and were unconnected. But in nineteenth century, Faraday and Maxwell along with others unified



these independent branches of physics. They proved that all three phenomena are manifestations of electromagnetic field.

- The simplest example is the electric field of an electric charge that exerts a force an another charge when it comes in the range. An electric current produces a magnetic field that exerts a force on magnetic materials.
- Such fields can travel through space, independent of charge and magnet, in the form of electromagnetic wave. The best example of electromagnetic wave is light. Finally, Einstein unified space, time and gravity in his theory of relativity.
- Quantum mechanics also unified two branches of science : physics and chemistry.
- In previous developments in physics, fundamental concepts were not different from those of everyday experience, such as particle, position, speed, mass, force, energy and even field. These concepts are referred as **Classical**.
- The world of atoms cannot be described and understood with these concepts. For atoms and molecules, the ideas and concepts used in dealing with objects in day to day life is not sufficient. Thus, it needed new concepts to understand the properties of atoms.
- A group of scientists W. Heisenberg, E. Schrodinger, P.A.M. Dirac, W. Pauli, M. Born and Neils Bohr, conceived and formulated these new ideas in the beginning of 20th century. This new formulation, a branch of physics, was named as **Quantum Mechanics**.

3.8.1 Limitations of Classical Mechanics

- The classical physics is complete and beautiful in explaining daily experiences where big bodies are involved. But it breaks down severely at subatomic level and failed to explain some of the phenomenon totally.
- The phenomena which classical physics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- In classical physics, a body which is very small in comparison with other body is termed as **Particle**. Whereas in quantum mechanics, the body which cannot be divided further is termed as **Particle**.
- The other main difference is the quantized energy state. In classical physics, an oscillating body can assume any possible energy. On the contrary, quantum mechanics says that it can have only discrete non-zero energy.

3.8.2 Need of Quantum Mechanics

- Classical mechanics successfully explained the motions of object which are observable directly or by instruments like microscope. But when classical mechanics is applied to the particles of atomic levels, it fails to explain actual behaviour. Therefore, the classical mechanics cannot be applied to atomic level, e.g. motion of an electron in an atom.
- Other phenomena which classical mechanics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- The above problems were solved by Max Planck in 1900 by the introduction of the formula

$$E = nh\nu$$

where, $n = 0, 1, 2, \ldots,$

h = Planck's constant = 6.63×10^{-34} J/s

- This is known as **Quantum Hypothesis** and marked the beginning of modern physics. The whole microscopic world obeys the above formula.

3.9 HEISENBERG'S UNCERTAINTY PRINCIPLE

[May 18]

- One of the tacit assumptions of classical physics, that the position of a mechanical system can be uniquely determined without disturbing its motion, is valid only for the motion of a body of ordinary size, like a cricket ball.
- But if one is considering the motion of an atomic particle, like an electron, a certain uncertainty is unavoidably introduced into the experimental measurement of its position and momentum.
- This uncertainty is not due to the imperfection of the measuring instruments but is something inherent in the nature of a moving body.
- The fact that a moving body must be regarded as a De Broglie wave group (packet) rather than as a localised entity suggests that, there is a fundamental limit to the accuracy with which we can measure its particle properties.

- A De Broglie wave group is shown in Fig. 3.7 (a). The particle may be anywhere within the group. For a very narrow wave group, as in Fig. 3.7 (b), the position of the particle can be readily found, but the wavelength λ, and hence the momentum $p = \frac{h}{\lambda}$, is impossible to establish.
- For a wide wave group, as in Fig. 3.7 (c), the wavelength and hence momentum estimate is satisfactory, but then the location of the position of the particle becomes uncertain.

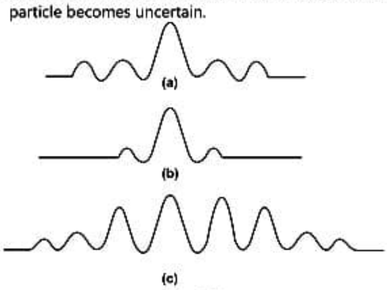


Fig. 3.7

- The answer to this question was given by Heisenberg in 1927, when he put forth the uncertainty principle.

Statement :

- Heisenberg's uncertainty principle states that, it is impossible to determine accurately and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables, like position x and momentum p; or energy E and time t, are called canonically conjugate variables.
- To examine the uncertainty principle, consider an electron of mass m associated with matter waves of wavelength λ. This electron can be found somewhere within this wave and therefore, the uncertainty in its position measurement Δx is equal to its wavelength λ.

$$\therefore \quad \Delta x = \lambda \qquad \text{... (3.16)}$$

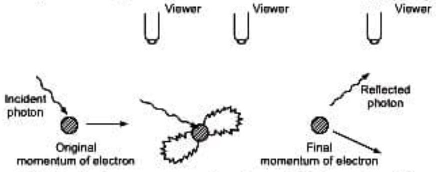


Fig. 3.8 : An electron cannot be observed without changing its momentum by an indeterminate amount

- To observe the electron we have to illuminate it with light, say of wavelength λ, as in Fig. 3.8. In this process, photons of light strike the electron and bounce off it. Each photon possesses the momentum $\frac{h}{\lambda}$ and when it collides with the electron, original momentum p of the electron is changed.
- The precise change of the momentum of the electron cannot be predicted, but it is likely to be of the same order of magnitude as the photon momentum $\frac{h}{\lambda}$. Thus, the electron cannot be observed without changing its momentum by an indeterminate amount.
- The act of measurement of its position introduces an uncertainty in its momentum Δp and this uncertainty in the momentum is at least equal to the momentum of incident photon.

$$\therefore \quad \Delta p = \frac{h}{\lambda} \qquad \text{... (3.17)}$$

From equations (3.16) and (3.17),

$$\Delta x \cdot \Delta p = \lambda \cdot \frac{h}{\lambda} = h \qquad \text{... (3.18)}$$

- It is clear from equation (3.18) that if the position of the electron is known exactly at any given instant, i.e. if $\Delta x = 0$, then the momentum becomes indeterminate and vice-versa. Thus, both the position and the momentum cannot be determined accurately and simultaneously.
- The product of uncertainty in position measurement Δx of a body at some instant and the uncertainty in its momentum measurement Δp at the same instant is at best equal to the Planck's constant h (more correctly $\frac{h}{4\pi}$).

i.e. $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

This is Heisenberg's uncertainty principle.

3.9.1 Uncertainty Principle Applied to the Pair of Variables

Energy and Time

- Kinetic energy and time form another pair of canonically conjugate variables. Consider again the Problem of an electron of mass m moving with velocity v. We can write the K.E. of the electron as

$$E = \frac{1}{2} mv^2 \qquad \text{... (3.19)}$$

- The uncertainty in the energy measurement ΔE can be found by differentiating equation (3.19), assuming mass to be constant.

$$\therefore \quad \Delta E = \frac{1}{2} m \cdot 2 v \Delta v$$

$$\Delta E = v (m \cdot \Delta v)$$

$$\Delta E = v \cdot \Delta p$$

$$\Delta E = \frac{\Delta x}{\Delta t} \cdot \Delta p \qquad \left(\because v = \frac{\Delta x}{\Delta t}\right)$$

Hence $\Delta E \cdot \Delta t = \Delta x \cdot \Delta p$

- But by Heisenberg's uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\therefore \quad \Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

- This means that the product of uncertainties in energy and time measurements is of the order of Planck's constant.

3.9.2 Illustration of Uncertainty Principle [May 19]

(1) Diffraction at a Single Slit

- Consider a narrow beam of electrons passing normally through a single vertical narrow slit of width Δy and producing a diffraction pattern on the screen. (See Fig. 3.9)

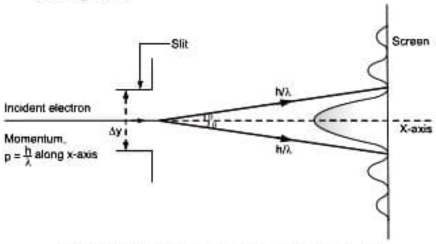


Fig. 3.9 : Diffraction of electrons at a single slit

- We know that the positions of minima in the diffraction pattern due to a slit of width a formed by incident light of wavelength λ are given by

$$a \sin\theta = n\lambda \qquad \text{... (3.20)}$$

where θ is the angle of deviation of n^th order minimum in the diffraction pattern.

- So if the first order minimum in the diffraction pattern due to a slit of width Δy is formed for an angle θ, when electron waves of wavelength λ are diffracted by it, we shall have

$$\Delta y \sin\theta = 1 \cdot \lambda \qquad \text{... (3.21)}$$

- All the electrons producing the diffraction pattern on the screen have passed through the slit, but we cannot say definitely at what position of the slit. So the uncertainty in position determination of electrons is equal to the width Δy of the slit, and from equation (3.21), we have

$$\Delta y = \frac{\lambda}{\sin\theta} \qquad \text{... (3.22)}$$

- Electrons are initially moving along positive x-axis. Their momentum along x-axis is $\frac{h}{\lambda}$ and they do not have any component of momentum along y-axis.
- But after diffraction at the slit, electrons are deviated from their initial path to form the diffraction pattern, and the y-component of their momentum may be between $\frac{h}{\lambda} \sin\theta$ and $-\frac{h}{\lambda} \sin\theta$ (See Fig. 3.9).
- So the uncertainty in momentum measurement along y - direction is given by

$$\Delta p_y = \frac{h}{\lambda} \sin\theta - \left(-\frac{h}{\lambda} \sin\theta\right)$$

$$\Delta p_y = \frac{2h}{\lambda} \sin\theta \qquad \text{... (3.23)}$$

From equations (3.21) and (3.22), we have

$$\Delta y \cdot \Delta p_y = \frac{\lambda}{\sin\theta} \cdot \frac{2h}{\lambda} \sin\theta = 2h$$

i.e. $\Delta y \cdot \Delta p_y \geq h$

- Thus, the product of uncertainties in position and momentum measurements of the electron is of the order of Planck's constant, which is Heisenberg's uncertainty principle.

(2) Why an Electron cannot Exist in the Nucleus [May 18]

- If the electrons had to exist inside the nucleus then its De-Broglie wavelength should be roughly of the order of nucleus diameter i.e. 10^{-14} m.

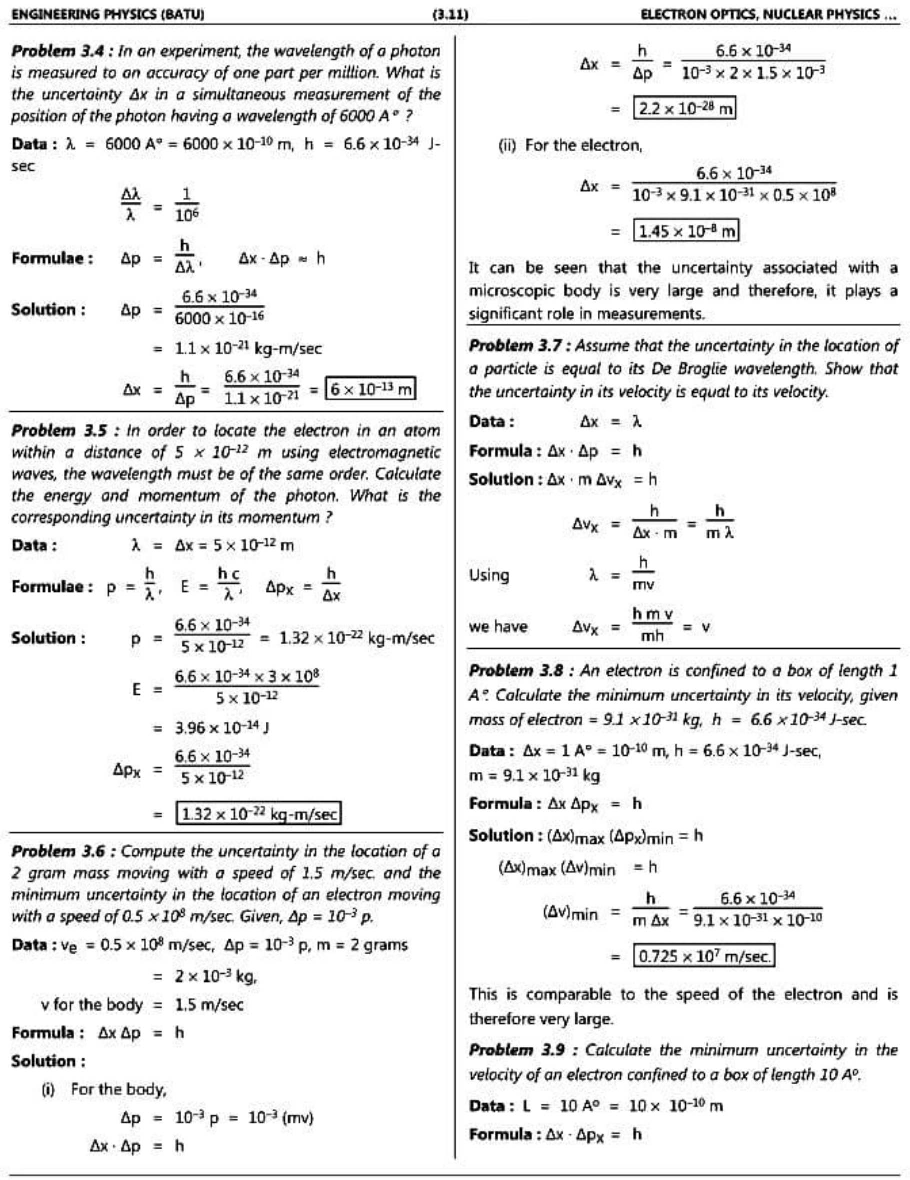
Therefore, the corresponding momentum will be

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}} = 6.63 \times 10^{-20} \text{ kg-m/sec}$$

$$\therefore \quad E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-9} \text{ J}$$

$$= \frac{2.42 \times 10^{-9}}{1.6 \times 10^{-19}} \text{ eV} = 15095 \text{ MeV}$$

- If the electron had to exist in the nucleus then its energy should be 15095 MeV. However, this is greater than the maximum binding energy of the nucleus. Thus, the electron cannot exist inside the nucleus.



Problem 3.4 : *In an experiment, the wavelength of a photon is measured to an accuracy of one part per million. What is the uncertainty Δx in a simultaneous measurement of the position of the photon having a wavelength of 6000 A° ?*

Data : $\lambda = 6000\ A^\circ = 6000 \times 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6}$$

Formulae : $\Delta p = \frac{h}{\Delta\lambda}, \quad \Delta x \cdot \Delta p \approx h$

Solution :

$$\Delta p = \frac{6.6 \times 10^{-34}}{6000 \times 10^{-16}}$$

$$= 1.1 \times 10^{-21} \text{ kg-m/sec}$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{1.1 \times 10^{-21}} = \boxed{6 \times 10^{-13} \text{ m}}$$

Problem 3.5 : *In order to locate the electron in an atom within a distance of 5×10^{-12} m using electromagnetic waves, the wavelength must be of the same order. Calculate the energy and momentum of the photon. What is the corresponding uncertainty in its momentum ?*

Data : $\lambda = \Delta x = 5 \times 10^{-12}$ m

Formulae : $p = \frac{h}{\lambda}, \quad E = \frac{hc}{\lambda}, \quad \Delta p_x = \frac{h}{\Delta x}$

Solution :

$$p = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}} = 1.32 \times 10^{-22} \text{ kg-m/sec}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-12}}$$

$$= 3.96 \times 10^{-14} \text{ J}$$

$$\Delta p_x = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}}$$

$$= \boxed{1.32 \times 10^{-22} \text{ kg-m/sec}}$$

Problem 3.6 : *Compute the uncertainty in the location of a 2 gram mass moving with a speed of 1.5 m/sec. and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/sec. Given, $\Delta p = 10^{-3}\ p$.*

Data : $v_e = 0.5 \times 10^8$ m/sec, $\Delta p = 10^{-3}$ p, m = 2 grams

$= 2 \times 10^{-3}$ kg,

v for the body = 1.5 m/sec

Formula : $\Delta x\, \Delta p = h$

Solution :

(i) For the body,

$$\Delta p = 10^{-3}\ p = 10^{-3}\ (mv)$$

$$\Delta x \cdot \Delta p = h$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{10^{-3} \times 2 \times 1.5 \times 10^{-3}}$$

$$= \boxed{2.2 \times 10^{-28} \text{ m}}$$

(ii) For the electron,

$$\Delta x = \frac{6.6 \times 10^{-34}}{10^{-3} \times 9.1 \times 10^{-31} \times 0.5 \times 10^8}$$

$$= \boxed{1.45 \times 10^{-8} \text{ m}}$$

It can be seen that the uncertainty associated with a microscopic body is very large and therefore, it plays a significant role in measurements.

Problem 3.7 : *Assume that the uncertainty in the location of a particle is equal to its De Broglie wavelength. Show that the uncertainty in its velocity is equal to its velocity.*

Data : $\Delta x = \lambda$

Formula : $\Delta x \cdot \Delta p = h$

Solution : $\Delta x \cdot m\, \Delta v_x = h$

$$\Delta v_x = \frac{h}{\Delta x \cdot m} = \frac{h}{m\lambda}$$

Using $\lambda = \frac{h}{mv}$

we have $\Delta v_x = \frac{hmv}{mh} = v$

Problem 3.8 : *An electron is confined to a box of length 1 A°. Calculate the minimum uncertainty in its velocity, given mass of electron = 9.1×10^{-31} kg, $h = 6.6 \times 10^{-34}$ J-sec.*

Data : $\Delta x = 1\ A^\circ = 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec,
$m = 9.1 \times 10^{-31}$ kg

Formula : $\Delta x\, \Delta p_x = h$

Solution : $(\Delta x)_{max}\, (\Delta p_x)_{min} = h$

$$(\Delta x)_{max}\, (\Delta v)_{min} = h$$

$$(\Delta v)_{min} = \frac{h}{m\,\Delta x} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}}$$

$$= \boxed{0.725 \times 10^7 \text{ m/sec.}}$$

This is comparable to the speed of the electron and is therefore very large.

Problem 3.9 : *Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A°.*

Data : L = 10 A° = 10×10^{-10} m

Formula : $\Delta x \cdot \Delta p_x = h$

Solution : $(\Delta x)_{max} (\Delta p_x)_{min} = h$

i.e. $(\Delta x)_{max}\, m(\Delta v_x)_{min} = h$

$$\Delta v_x = \frac{h}{m \cdot (\Delta x)_{max}} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-9}}$$

$$= \boxed{0.725 \times 10^6 \text{ m/sec.}}$$

***Problem 3.10 :** An electron has a speed of 600 m/sec with an accuracy of 0.005 %. Calculate the uncertainty with which we can locate the position of the electron.*

Data :

$$v = 600 \text{ m/sec}$$

$$\Delta v = 0.005 \text{ \% of } v$$

$$= \frac{0.005}{100} \times 600 \text{ m/sec}$$

Formula : $\Delta x \cdot \Delta p = h$

$$\Delta x = \frac{h}{\Delta p} = \frac{h}{m\, \Delta v}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31}} \times \frac{0.005}{100} \times 600$$

$$= \boxed{0.024 \text{ m}}$$

3.10 PHYSICAL SIGNIFICANCE OF WAVE FUNCTION

3.10.1 Concept of Wave Function

- A wave motion appears in almost all branches of physics. A wave motion is defined as a periodic disturbance travelling with finite velocity through a medium or space.
- The simplest form of vibration is simple harmonic motion (S.H.M.) and a particle executing S.H.M. acts as a source which radiates waves.
- The wave motion provides a way for energy and momentum to move from one place to another without material particles making that journey.
- The waves can be classified according to their broad physical properties into mainly three categories :
  - Electromagnetic waves which need not require any medium to propagate.
  - Matter waves which give the probability amplitude of finding a particle at a given position and time.
  - **Mechanical Waves :** The mechanical waves are simplest one to understand because they are produced by some sort of mechanical vibrations which we can see.
- When a mechanical wave passes through a medium, the medium particles perform an S.H.M. given by equation

$$y = A \cos \omega t \qquad \text{... (3.24)}$$

where A is the amplitude of the oscillation and $\omega = 2\pi\upsilon$, where υ is the frequency.

  - This equation is applicable to all individual particles affected by the wave. Suppose the wave is progressing forward with velocity v. If P is the origin of the wave, then a particle at Q at a distance x from P will receive the wave x/v sec later than P did.

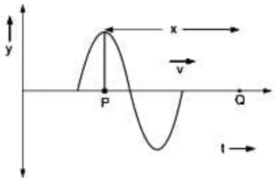


Fig. 3.10 : Progressive wave moving with velocity v

- Hence, its displacement at time t and distance x from the origin will be

$$y = A \cos \omega \left(t - \frac{x}{v}\right) \qquad \text{... (3.25)}$$

The wave equation of such a wave is

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \qquad \text{... (3.26)}$$

The solution of equation (3.26) is given by

$$y = Ae^{-i\omega (t - x/v)} \qquad \text{... (3.27)}$$

- In a string we can represent the wave disturbance by the transverse displacement of y. Similarly, for light waves the field vectors E and B vary in space and time, for sound waves, pressure P varies in space and time. In the same way, for matter waves, the wave function ψ varies in space and time.
- So ψ in wave mechanics is analogous to electric field E in electromagnetic waves or to pressure P in the sound waves. However, ψ itself unlike E and P has no direct physical significance, but gives a measure of the probability of finding a particle at a particular position. Hence, it is called **Probability Amplitude**.
- However, a probability is always real and positive, whereas ψ can be positive or negative. Therefore, ψ^2 is taken which is always positive. In general, ψ is complex,

therefore, one takes $|\psi|^2$ instead of ψ^2, where $|\psi|^2 = \psi^* \psi$, ψ^* denoting the complex conjugate of ψ. In any case, $\psi^* \psi$ is always real and positive.

- If dv is a volume element located at a point, then the probability of finding the particle in the volume element at time t is proportional to $\psi^*\psi dv$. By analogy with ordinary mass density, the square of the wave function $\psi^*\psi$ is called the **Probability Density** i.e. **Probability Per Unit Volume**.

3.10.2 Physical Significance of the Wave Function

- Schrodinger interpreted ψ in terms of charge density. If A is the amplitude of an electromagnetic wave, then the energy per unit volume, i.e. energy density is equal to A^2. Also, the photon energy $h\upsilon$ is constant. So the number of photons per unit volume, i.e. the photon density is equal to $\frac{A^2}{h\upsilon}$, and it is proportional to the amplitude square.
- Similarly, if ψ is the amplitude of matter waves at any point in space, the particle density at that point may be taken as proportional to $|\psi|^2$. So $|\psi|^2$ is a measure of particle density and on multiplying this by the charge of the particle, we shall get the charge density. Thus, $|\psi|^2$ is a measure of **Charge Density**.
- According to Max Born, the value of $|\psi|^2$ at a point at a given time is related to the probability of finding the body described by its wave function ψ at that point at that instant. A large value of $|\psi|^2$ means a strong possibility of the presence of the body, while a small value of $|\psi|^2$ means a slight possibility of its presence. As long as $|\psi|^2$ is not actually zero somewhere, there is a definite chance, however small, of detecting the body there.
- Although the wave function ψ of a particle is spread out in space, this does not mean that the particle itself is also thus spread out. When an experiment is performed to detect a particle, an electron for instance, a whole electron is either found at a certain place and time, or it is not. There is nothing like 20 % of an electron. However, it is certainly possible that there is 20 % chance that the electron be found at that place and time, and it's likelihood that is specified by $|\psi|^2$ or $\psi\psi^*$, ψ^* being the complex conjugate of ψ.
- $|\psi|^2$ or $\psi\psi^*$ is taken as the probability density, i.e. the probability of finding the particle in unit volume. So the probability of the particle being present in a volume element dx·dy·dz is $|\psi|^2$ dx dy dz. Then, the wave function ψ is called the **Probability Density Amplitude**.
- Since the particle is certainly to be found somewhere in space, we must have,

$$\iiint |\psi|^2 \, dx \, dy \, dz = 1 \qquad \text{... (3.28)}$$

the triple integral extending over all possible values of x, y, z.

- A function ψ satisfying this relation is called a **Normalised Wave Function** and equation (3.28) is known as the **Normalisation Condition**. Thus, ψ has to be a normalisable function.

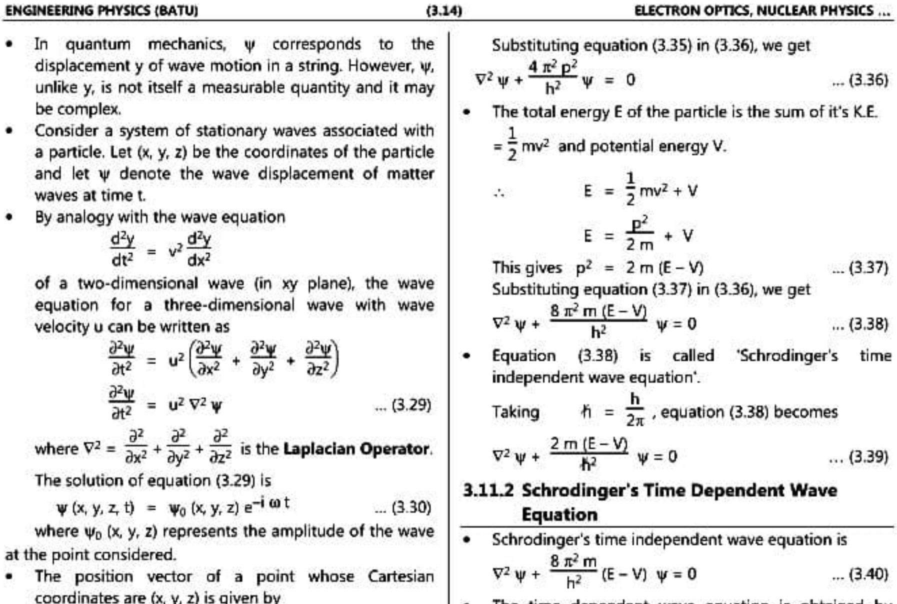
  Besides being normalisable, ψ must also satisfy the following conditions :
  - Must be a single valued function, because ψ is related to the probability of finding the particle at a given place and time, and the probability can have only one value at a given point and time.
  - Must be finite, because the particle exists somewhere in space, and so integral over all space must be finite.
  - And its derivatives $\frac{\partial \psi}{\partial x}$, $\frac{\partial \psi}{\partial y}$, $\frac{\partial \psi}{\partial z}$ must be continuous everywhere in the region where ψ is defined.

3.11 SCHRODINGER'S WAVE EQUATION

- Schrodinger started with De Broglie's idea of matter waves and developed it into a mathematical theory known as **Wave Mechanics**. Schrodinger's wave equation is the mathematical representation of matter waves associated with a moving particle. There are two types of Schrodinger's wave equations :
  1. Schrodinger's time independent wave equation
  2. Schrodinger's time dependent wave equation.

3.11.1 Schrodinger's Time Independent Wave Equation [Dec. 17, 18, May 19]

- According to De Broglie's theory, a particle of mass m moving with a velocity v has a wave system of some kind associated with it, and its wavelength is given by $\lambda = \frac{h}{mv}$. The waves are produced only when something oscillates. Though we do not know the quantity that vibrates to produce the matter waves, but we can indicate that quantity by ψ.
- The periodic changes in ψ produce the wave system associated with the particle, just as the periodic changes in the displacement y of a string produce a wave system along the string.



- In quantum mechanics, ψ corresponds to the displacement y of wave motion in a string. However, ψ, unlike y, is not itself a measurable quantity and it may be complex.
- Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of matter waves at time t.
- By analogy with the wave equation

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

of a two-dimensional wave (in xy plane), the wave equation for a three-dimensional wave with wave velocity u can be written as

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = u^2 \nabla^2 \psi \qquad \text{... (3.29)}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the **Laplacian Operator**.

The solution of equation (3.29) is

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} \qquad \text{... (3.30)}$$

where ψ_0 (x, y, z) represents the amplitude of the wave at the point considered.

- The position vector of a point whose Cartesian coordinates are (x, y, z) is given by

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ being unit vectors along the axes. So equation (3.30) can be written as

$$\psi(\vec{r}, t) = \psi_0(\vec{r})\, e^{-i\omega t} \qquad \text{... (3.31)}$$

Differentiating equation (3.31) twice with respect to time t, we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_o(r)\, e^{-i\omega t}$$

and $$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)^2 \psi_0(\vec{r})\, e^{-i\omega t}$$

$$= -\omega^2\psi \qquad \text{... (3.32)}$$

From equations (3.29) and (3.32), we get

$$u^2\nabla^2\psi = -\omega^2\psi$$

$$\therefore \nabla^2\psi + \frac{\omega^2}{u^2}\psi = 0 \qquad \text{... (3.33)}$$

But $\omega = 2\pi\upsilon$, and $u = \upsilon\lambda$

$\therefore$ Equation (3.33) becomes

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \text{... (3.34)}$$

- The De Broglie wavelength of the waves associated with the particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad \text{... (3.35)}$$

Substituting equation (3.35) in (3.36), we get

$$\nabla^2\psi + \frac{4\pi^2 p^2}{h^2}\psi = 0 \qquad \text{... (3.36)}$$

- The total energy E of the particle is the sum of it's K.E. $= \frac{1}{2}mv^2$ and potential energy V.

$$\therefore \quad E = \frac{1}{2}mv^2 + V$$

$$E = \frac{p^2}{2m} + V$$

This gives $p^2 = 2m(E - V)$ ... (3.37)

Substituting equation (3.37) in (3.36), we get

$$\nabla^2\psi + \frac{8\pi^2 m(E - V)}{h^2}\psi = 0 \qquad \text{... (3.38)}$$

- Equation (3.38) is called 'Schrodinger's time independent wave equation'.

Taking $\hbar = \frac{h}{2\pi}$, equation (3.38) becomes

$$\nabla^2\psi + \frac{2m(E - V)}{\hbar^2}\psi = 0 \qquad \text{... (3.39)}$$

3.11.2 Schrodinger's Time Dependent Wave Equation

- Schrodinger's time independent wave equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \text{... (3.40)}$$

- The time dependent wave equation is obtained by eliminating E from the time independent equation. Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of the matter waves at time t. If u be the wave velocity, then the equation for a three-dimensional wave motion can be written as,

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = u^2\nabla^2\psi \qquad \text{... (3.41)}$$

where $$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

is the '**Laplacian Operator**'

The solution of equation (3.41) is

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} = \psi_0(\vec{r})\, e^{-i\omega t} \qquad \text{... (3.42)}$$

where ψ_0 (x, y, z) is the amplitude of the wave at the point considered.

- Differentiating equation (3.42) with respect to time t, we get

$$\frac{\partial\psi}{\partial t} = (-i\omega)\psi_0(\vec{r})\, e^{-i\omega t} = -i\omega\psi \qquad \text{... (3.43)}$$

Now, $\omega = 2\pi\upsilon$ and $E = h\upsilon$ or $\upsilon = \frac{E}{h}$

$$\therefore \quad \omega = \frac{2\pi E}{h}$$

Putting this value of ω in equation (3.43), we get

$$\frac{\partial \psi}{\partial t} = -i\,\frac{2\pi E}{h}\,\psi \qquad \text{... (3.44)}$$

Multiplying both sides of equation (3.44) by i,

$$i\frac{\partial \psi}{\partial t} = i^2\frac{2\pi}{h}\,E\,\psi$$

$$\therefore \qquad E\,\psi = i\frac{h}{2\pi}\frac{\partial \psi}{\partial t} = i\hbar\,\frac{\partial \psi}{\partial t} \qquad \text{... (3.45)}$$

From equations (3.40) and (3.45), we get

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}\left(\frac{i\,h}{2\pi}\frac{\partial \psi}{\partial t} - V\,\psi\right) = 0$$

Multiplying both sides of this equation by $\frac{-h^2}{8\pi^2 m}$, we get

$$-\frac{h^2}{8\pi^2 m}\nabla^2\psi - \frac{i\,h}{2\pi}\frac{\partial \psi}{\partial t} + V\,\psi = 0$$

$$\left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{i\,h}{2\pi}\frac{\partial \psi}{\partial t} \qquad \text{... (3.46)}$$

$$\text{or}\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = i\,\hbar\,\frac{\partial \psi}{\partial t} \qquad \text{... (3.47)}$$

Equation (3.47) is called 'Schrodinger's time dependent wave equation'.

$$\text{Taking} \qquad H = \left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$$

as **Hamiltonian Operator**

$$\text{and} \qquad E = \frac{i\,h}{2\pi}\frac{\partial}{\partial t} = i\,\hbar\frac{\partial}{\partial t}$$ as **Eigen Operator**,

equation (3.47) becomes

$$H\,\psi = E\,\psi \qquad \text{... (3.48)}$$

SUMMARY

- e/m of an electron can be measured by Thomson's method.
- By using Millikan's method charge of an electron can be measured.
- **Bainbridge Mass Spectrograph** is a device used to (i) Detect the presence of isotopes, (ii) Find an accurate value of isotopic masses, (iii) Find their abundance.
- **Atomic Nucleus** occupies a very small volume with a diameter of 10^{-14} m. It consists of Z protons and A-Z neutrons. Proton and neutrons are collectively called as nucleons.
- **Nuclear Force :** Nucleons are held in the nucleus through short range nuclear forces.
- **A M U :** Masses of nucleons and nuclei are expressed in atomic mass units (amu)

  $$1\text{ amu} = 1.67 \times 10^{-27}\text{ kg}$$
- **Mass Defect** of a nucleus is the difference between its theoretical mass and its actual mass.
- A GM counter can be used for detecting α, β and γ radiations.
- **Heisenberg's Uncertainty Principle :** It is impossible to determine precisely and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables are called as canonically conjugate variables.

  $$\Delta x \cdot \Delta p \geq h$$

  (Heisenberg's uncertainty principle for position and momentum.)

  $$\Delta E \cdot \Delta t \geq h$$

  (Heisenberg's uncertainty principle for energy and time.)
- **Schrodinger's Wave Equation :** It is the mathematical representation of matter waves associated with a moving particle. They are of two types :

  (i) Schrodinger's time independent wave equation :

  $$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\,\psi = 0$$

  (ii) Schrodinger's time dependent wave equation :

  $$\left(\frac{-h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t}$$

  i.e. $\quad H\psi = E\psi$
- **Wave Function :** The variable quantity characterizing De Broglie waves is denoted by ψ and it is called the wave function of the particle. This wave function contains all the information about the particle.
- **Probability Density :** The quantity $|\psi(x, y, z, t)|^2$, called the probability density or probability distribution function, determines the probability in unit volume of finding a particle at a given position at a given time.
- **The Probability** of a particle being present in a volume element dx·dy·dz is $|\psi|^2 \cdot dx\,dy\,dz$.

  Normalization condition

  The probability of finding the particle in all space is

  $$\underset{\text{all space}}{\iiint} |\psi|^2\,dx\,dy\,dz = 1.$$

  This is the normalization condition. A wave function ψ satisfying this relation is called a normalized wave function.
- **The Wave Function** should satisfy the following conditions :
  - (i) It should be a normalized function.
  - (ii) It should be a well behaved function i.e., single valued, finite and continuous.

IMPORTANT FORMULAE

- **e/m by Thomson's Method :**

  $$\frac{e}{m} = \frac{E}{B^2}\cdot\frac{d}{ED \times OG}$$
- **Millikan's Oil Drop Method :**

  $$q = \frac{\sigma\,\pi\,\eta}{E}\left(\frac{g\,\eta\,v1}{2g\,(\rho - \sigma)}\right)^{1/2}(v_1 - v_2)$$

- **Bainbridge Mass Spectrograph :** The charge to mass ratio is given by,

$$\frac{q}{m} = \frac{E}{B \cdot B'} \cdot \frac{1}{r}$$

- Heisenberg's Uncertainty Principle,
  For position and momentum

$$\Delta x \cdot \Delta p \geq h,$$

  For energy and time,

$$\Delta E \cdot \Delta t \geq h$$

- Normalization condition $\iiint_{-\infty}^{+\infty} \psi \psi^* \, dx \, dy \, dz = 1.$

- Schrodinger's wave equation,
  Time independent

$$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

  Time independent

$$\left(-\frac{h^2}{8\pi^2 m} \nabla^2 + V\right) \psi = \frac{ih}{2\pi} \frac{\partial \psi}{\partial t} ; \quad H\psi = E\psi$$

UNSOLVED PROBLEMS

1. Two beams of U^{235} and U^{238} are separated by a 180° magnetic focussing (as in a mass spectrograph). Find the separation of the beams at the focus, if their velocities are equal and the U^{238} beam has a radius of 1 m in a field of 1 weber/sqm.
   [Ans. Beam separation = 0.0255 m]
2. The electric field between the plates of a velocity selector in a Bainbridge mass spectrograph is 100 kV/m and the magnetic induction in both magnetic fields is 0.6 wb/m². A stream of singly charged neons moves in a circular path of radius 6 cm in the magnetic field. Find the mass number of the neon isotope. **[Ans.** m = 21]
3. The average time that an atom retains excess excitation energy before emitting it as electromagnetic radiation is 10^{-8} sec. Calculate the limit of accuracy with which the excitation energy of the emitted radiation can be determined.
   [Ans. 6.63×10^{-26} J]

EXERCISE

1. Explain with a neat diagram the principle and working of a Bainbridge Mass Spectrograph.
2. Explain use of Bainbridge Mass Spectrograph for detection of isotopes.
3. Give principle construction and working of GM counter.
4. Write a short note on Heisenberg's uncertainty principle.
5. State Heisenberg's uncertainty principle and explain it using the concept of De-Broglie wave groups.
6. Explain Thomson's method for measurement of e/m.
7. How charge of an electron can be measured by Millikan's method.
8. Derive Schrodinger's time dependent and time independent wave equation.
9. Explain the physical significance of ψ, ψ^2.
10. State and explain the principle of uncertainty and illustrate it by an experiment on diffraction at a single slit.
11. State and explain Heisenberg's uncertainty principle. Illustrate it by the experiment for location of a particle by microscope.

REFERENCES

Animation of single slit diffraction pattern :
http://www.walter-fendt.de/ph14e/singleslit/htm

An interactive animation of diffraction pattern with a grating :
http://www.physics.uq.edu.au/people/mcintype/php/laboratories/index.php?e=14.

More information about resolving power :
http://www.astronomynotes.com/telescop/s6.htm

More information about X-ray diffraction :
http://www.eserc.stonybrook.edu/ProjectJava/Bragg/

UNIVERSITY QUESTIONS

December 2017

1. With neat diagram explain principle and working of Bainbridge Mass Spectrograph. **[6]**
2. Derive the time independent Schrodinger's wave equation. **[6]**

May 2018

1. State Heisenberg's Uncertainty Principle and prove that electron cannot exist in the nucleus. **[6]**

December 2018

1. Discuss Thomson's method of determination of e/m of an electron. **[6]**
2. Derive time independant Schrodingers wave equation. **[6]**

May 2019

1. Explain with diagram principle and working of Bainbridge mass spectrograph. **[6]**
2. Show that electron does not exists inside the nucleus, with the help of Heisenberg's uncertainty principle. **[3]**
3. Derive Schrodinger's time dependent wave equation. **[6]**

✠ ✠ ✠

UNIT IV

CRYSTAL STRUCTURE, X-RAYS AND ELECTRODYNAMICS

4.1 INTRODUCTION TO CRYSTAL STRUCTURE AND X-RAYS

- The atomic nature of matter has now been established. Nature, however, is never perceived in the atomic form but manifests itself in either of the following forms : (i) Solid, (ii) Liquid, (iii) Gas, (iv) Plasma. The first three occur very commonly while plasma is assumed at elevated temperatures.
- Solid, liquid and gaseous states can be differentiated, superficially, upon certain features exhibited by them. For example, solids possess definite shape, structure and volume, while liquids are devoid of a definite shape but possess a definite volume. Gases possess neither definite shape nor volume.
- It can also be seen that the forms can be differentiated on the basis of their interatomic distance. In the solid state, distances are of the order of a few Angstroms and there is some kind of order, whereas in liquid state there is a transition to a less ordered state. In gaseous state, there is extreme disorder.
- When the atoms or molecules in a solid are arranged in a regular fashion then it is known as **Crystalline**, otherwise it is **Amorphous**. A crystal is a solid composed of a periodic array of atoms. Actual materials are composed of an aggregate of single crystals.
- The study of **Solid State Physics** aims at interpreting the macroscopic properties in terms of the microscopic properties of the minute particles of which the crystal is composed. The study of the geometric forms and physical properties of crystalline solids using X-rays, electron neutron beams etc. constitute the science of **Crystallography**.

4.2 UNIT CELL

4.2.1 Space Lattice

- A **Space Lattice** or a **Lattice** is defined as a regular three dimensional periodically repetitive arrangement of points in space, which is infinite in extent.
- Consider an array of points in such a way that the environment around any one point is identical with the environment about any other point. Such an array of points in two dimensions (See Fig. 4.1) is called the **Plane Lattice**.
- In order to construct the two dimensional lattice, choose any two convenient axes OA and OB. The points lie at equal intervals say 'a' along OA and 'b' along OB.

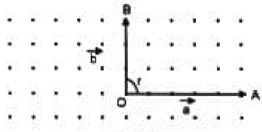


Fig. 4.1

- On extending this array of points in three dimensions, we get a space lattice. The points are arranged at equal intervals say 'c' in the third direction.
- The position vector of any lattice point in two dimensional lattice by choosing any lattice point as origin is

$$\overline{T} = n_1 \overline{a} + n_2 \overline{b}$$

Here n_1, n_2 are integral values representing the number of lattice points along OA and OB and $\overline{a}$ and $\overline{b}$ are called the primitive vectors. $\overline{T}$ is a translation vector connecting two lattice points.

- Similarly $\overline{T}$ in three dimensions is expressed as

$$\overline{T} = n_1 \overline{a} + n_2 \overline{b} + n_3 \overline{c}$$

4.2.2 Unit Cell

[May 19]

- The parallelopiped formed by using the translations $\overline{a}$, $\overline{b}$, $\overline{c}$ as edges is called the **Unit Cell** of the space lattice or the **Primitive Cell**.

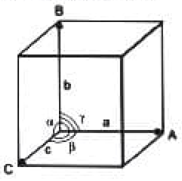


Fig. 4.2 : Unit cell in space lattice

- The angles between primitives (b, c), (c, a) and (a, b) are denoted by α, β, and γ.
- Given a lattice work several unit cells are possible. All primitive cells have equal volume which is the minimum possible volume among all unit cells.
- Thus, a **Lattice** is defined as a parallel arrangement of points provided the environment about any point is identical with the environment about any other point.
- A **Non-Primitive Unit** cell contains additional lattice points, either on face or within the unit cell.
- There are two distinct type of unit cells, **Primitive** and **Non-Primitive**.
- The primitive unit cells contain only one lattice point, while non-primitive unit cell contain additional points. The additional lattice point may be in face of the unit cell or within the unit cell.
- The number of additional lattice point per unit cell may be more than one.
- The Fig. 4.3 show the primitive and non-primitive unit cell.

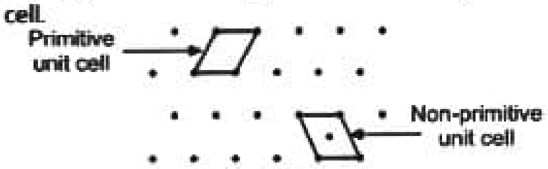


Fig. 4.3

4.3 CUBIC SYSTEMS

- The crystals are classified into seven systems on the basis of the shape of unit cell as given in the Table 4.1.

Table 4.1

No.	Name of the Crystal System	Lattice Symbol	Number of Different Lattices in the System	Relation of Axes and the Angles Defining the Cells
1.	Cubic	P (simple cubic) I (Body centred) F (Face centred)	3	$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$
2.	Tetragonal	P, I	2	$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
3.	Hexagonal	P	1	$a = b \neq c$ $\alpha = \beta = 90^\circ,\ \gamma = 120^\circ$
4.	Trigonal	R (rhombohedron)	1	$a = b = c$ $\alpha = \beta = \gamma < 120^\circ, \neq 90^\circ$
5.	Ortho-rhombic	P, I, F C (base centred)	4	$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
6.	Monoclinic	P, C	2	$a \neq b \neq c$ $\alpha = \gamma = 90^\circ, \neq \beta$
7.	Triclinic	P	1	$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma$
		Total	14	

- The cubic system is the simplest and the commonest while the triclinic crystals are least symmetrical.

For a cubic system, there exist three types of lattices :

1. There is one lattice point at each of the eight corners of the unit cell. This type is called the **Primitive** or **Simple Cubic Cell (P)** of the system.
2. There is one lattice point at each of the eight corners and one lattice point at the centre of the cubic cell. This is a body **Centred Cubic Cell (I)** (bcc type).
3. There is one lattice point at each of the eight corners and one lattice point at the centres of each of the six faces of the cubic cell. This is a **Face Centred Cubic Cell (F)** (fcc type) (See Fig. 4.4).

The corresponding Bravais lattices are displayed below in Fig. 4.4.

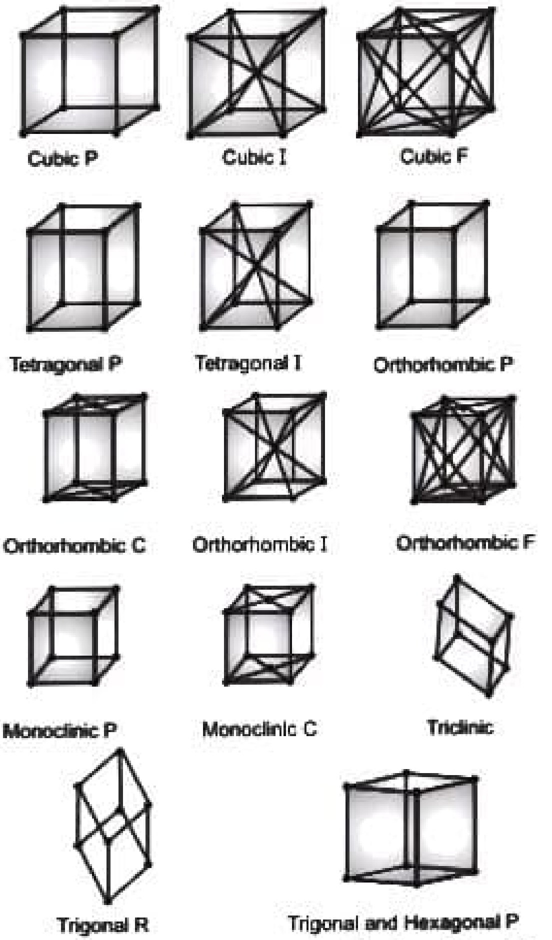


Fig. 4.4 : Bravais space lattice types in three dimensions

4.5 NUMBER OF ATOMS PER UNIT CELL

- As each lattice point is shared by eight cubes, we have $\frac{1^{th}}{8}$ of a point (of an atom situated at the lattice point) contributing to the cell. The 8 such corners contribute 8 × 1/8 = 1. On repeating the lattice points by identical atoms, this structure therefore has **One Atom per Unit Cell.**

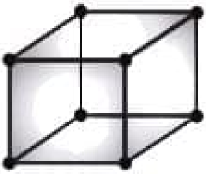


Fig. 4.5 : Simple cubic primitive cell

2. **Body Centred Cubic (BCC) Lattice :**

- As shown in Fig. 4.6 there are eight atoms at the corners and one at the body centre.
- The central atom is totally shared by the unit cell and the contribution due to the 8 corner atoms is $\frac{1}{8}$ each giving 1 atom in total. Hence, there are **Total 2 Atoms per Unit Cell** for this structure. Alkali metals like Li, Na, K occur naturally in this form. See Fig. 4.6.

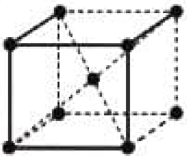


Fig. 4.6 : BCC unit cell

3. **Face Centred Cubic Lattice (FCC) :**

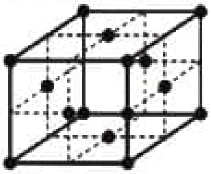


Fig. 4.7 : FCC unit cell

- Each unit cell consists of eight corner atoms each being a member of eight cells. In addition to this there are atoms situated at the centres of the six faces of the cube. Each face is shared by two adjoining cubes.
- Hence there are $\frac{8}{6} + \frac{6}{2}$ = 4 **Atoms per Unit Cell** for this lattice.

  Noble metals like gold, silver and copper occur in this group.

4.6 CO-ORDINATION NUMBER

- The co-ordination number is defined as the number of nearest neighbours to a given atom in a crystal lattice.

1. **Simple Cubic Lattice :**

- This crystal has one atom at each of the eight corners of the cube. Taking the atom at one corner as the origin and the X, Y, Z axes along the three edges passing through that corner, the positions of nearest neighbours are $\pm a\hat{i}, \pm a\hat{j}, \pm a\hat{k}$. i, j, k being unit vectors along X, Y and Z axes.
- The atoms nearest to the origin having co-ordinates (± a, 0, 0), (0 ± a, 0), (0, 0, ± a) are six in number. Hence co-ordination number of a simple cubic lattice is 6. The distance between two neighbours is 'a'.

2. **BCC Lattice :**

- In this lattice there is one atom at each corner of the cube and one atom at the body centre. Taking the central atom as the origin, the positions of nearest neighbours are $\left((\pm \frac{a}{2}\hat{i}, \pm \frac{a}{2}\hat{j}, \pm \frac{a}{2}\hat{k})\right)$
- The nearest neighbours are eight in number. Hence co-ordination number of BCC lattice is 8. The distance between the two nearest neighbours is

$$\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = a\sqrt{\frac{3}{2}}$$

(half the diagonal of the cube)

3. **FCC Lattice :**

- In this lattice, there is one atom at each of the eight corners of the cube and one atom at the centre of each of the six faces of the cube. Taking any one of the lattice points as the origin, the positions of the nearest neighbours are $\left(\pm \frac{a}{2}\hat{i}, \pm \frac{a}{2}\hat{j}\right), \left(\pm \frac{a}{2}\hat{j}, \pm \frac{a}{2}\hat{k}\right), \left(\pm \frac{a}{2}\hat{k}, \pm \frac{a}{2}\hat{i}\right)$
- The atoms nearest to the origin are 12 in number. Hence co-ordination number of FCC lattice is 12.
- The distance between two nearest neighbours (of the same kind of ions) is

$$\sqrt{\left(\frac{a}{2}\right)^2 + (0)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

The distance between the centres of two nearest neighbouring atoms is called as **Nearest Neighbour Distance**. It is denoted by 'a'.

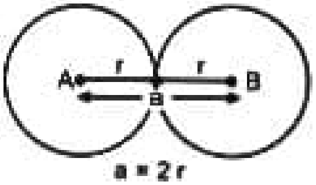


Fig. 4.8 : Nearest neighbour distance

If 'r' is considered to be radius of atom, then a = 2r i.e. the nearest neighbour distance is twice of the radius of atoms.

Let us calculate the radii of different Bravais lattices of a cubical crystal.

(i) Simple Cube Cell :

A simple cube cell in two-dimensional space can be represented as -

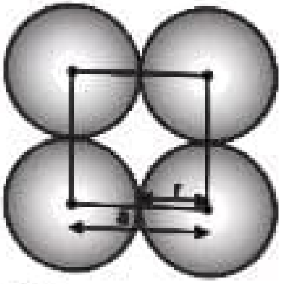


Fig. 4.9 : Radius of simple cube unit cell

$$a = 2r$$

$$\boxed{r = \frac{a}{2}}$$

Hence for a simple or primitive cube unit cell, the radius of the atom is half of the distance between centres of the two nearest neighbouring atoms.

(ii) Face Centered Cubic Cell (FCC) :

$$r^2 = \frac{2a^2}{16}$$

$$\therefore \quad \boxed{r = \frac{a\sqrt{2}}{4}}$$

(iii) Body Centered Cubic Cell (BCC) :

In a body centered cubical unit cell, one of the atoms is present inside the unit cell, so it can be shown in Fig. 4.11. It should be noted that the line BC passes through the atom that is present at the centre of the unit cell.

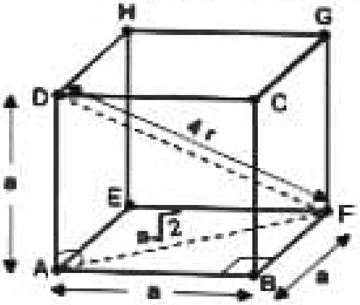


Fig. 4.11 : Radius of body centered cubical unit cell

Consider right angled Δ ABF, ∠ ABF = 90°

$$\therefore \quad AF^2 = AB^2 + BF^2$$

$$= a^2 + a^2$$

$$= 2a^2$$

$$AF = a\sqrt{2} \qquad \text{... (4.1)}$$

Consider right angled Δ DAF, ∠ DAF = 90°

$$\therefore \quad DF^2 = AD^2 + AF^2 \qquad \text{... (4.2)}$$

Substituting equation (4.1) in (4.2),

$$(4r)^2 = a^2 + (a\sqrt{2})^2$$

$$16r^2 = a^2 + 2a^2$$

$$16r^2 = 3a^2$$

$$\boxed{r = \frac{a\sqrt{3}}{4}}$$

4.8 PACKING DENSITY

Side of the unit cell 'a' = 2r

Volume of the unit cell = a^3

$$\therefore \quad \text{Packing density} = \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{\frac{4}{3}\pi r^3}{(2r)^3} = \frac{\pi}{6}$$

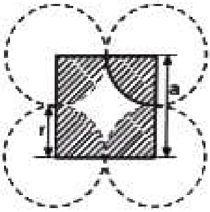


Fig. 4.12 : Simple cubic lattice

2. Body Centred Cube :

Number of atoms per unit cell = 2

Volume of two atoms = $2 \times \frac{4}{3}\pi r^3$

Side of the unit cell 'a' = $\frac{4r}{\sqrt{3}}$

Volume of the unit cell = a^3

$$\therefore \quad \text{Packing density} = 2 \times \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{\sqrt{3}\pi}{8}$$

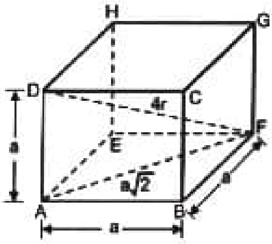


Fig. 4.13 : BCC lattice

3. Face Centred Cube :

Number of atoms per unit cell = 4

Volume of 4 atoms = $4 \times \frac{4}{3}\pi r^3$

Side of the unit cell 'a' = $\frac{4r}{\sqrt{2}}$

Volume of the unit cell = a^3

$$\therefore \quad \text{Packing fraction} = \frac{4 \times \frac{4}{3}\pi r^3}{\left(\frac{4r}{\sqrt{2}}\right)^3} = \sqrt{2}\,\frac{\pi}{6}$$

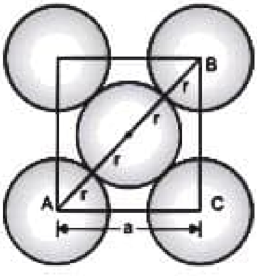


Fig. 4.14 : FCC lattice

4.9 RELATION BETWEEN LATTICE CONSTANT AND DENSITY [May 18]

- Consider a cubic lattice of lattice constant 'a'.

  If ρ is the density of the crystal then,

  $\therefore$ Mass in each unit cell = $a^3\rho$ ... (4.3)

  where a^3 = volume of unit cell
- If M is the molecular weight, N the Avogadro number (the number of molecules per kg. mole of the substance) then mass of each molecule = $\frac{M}{N}$
- If n is the number of molecules (lattice points) per unit cell, then

  $$\text{Mass in each unit cell} = \frac{nM}{N} \qquad \text{... (4.4)}$$

  From (4.3) and (4.4), $a^3\rho = \frac{nM}{N}$

  $$\text{Or} \qquad a = \left(\frac{nM}{N\rho}\right)^{1/3} \qquad \text{... (4.5)}$$

  From this relation, the lattice constant 'a' can be calculated.

4.10 LATTICE PLANES AND MILLER INDICES

- A crystal lattice can be considered as an aggregate of a set of parallel equidistant planes passing through the lattice points, which are known as **Lattice Planes**.
- For a particular lattice, these planes may be chosen in different ways as shown, for example, in Fig. 4.15. The problem is that how to designate these planes in crystal Miller developed a method to designate a set of parallel planes in a crystal by three numbers (h, k, *l*) known as **Miller Indices**.

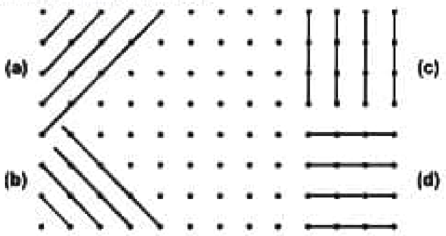


Fig. 4.15

- The steps in determination of Miller indices is given below :
  - ➢ Consider that the intercepts by the given lattice plane on the three crystal axes X, Y and Z are in the ratio pa : qb : rc where a, b, c are the primitives or lattice constants along X, Y, Z axes. p, q, r may be either small integers or simple fractions.
  - ➢ Take the reciprocals of p, q, r i.e. $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$.
  - ➢ Determine the smallest possible integers h, k, l such that

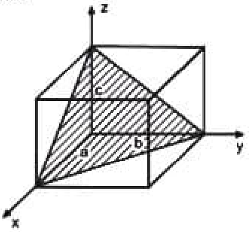


Fig. 4.16

$$h:k:l = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}$$

where $h = \frac{L}{p}, K = \frac{L}{q}, l = \frac{L}{r}$

and L is the L.C.M. of p, q, r

- The numbers h, k, l are called the **Miller Indices** of a given set of planes and the plane is specified by (h k l).

To Illustrate the Concept of Miller Indices : Let a given plane cut intercepts on the axes $\overline{a}, \overline{b}, \overline{c}$ as 3a, 6b and 8c.

Then
$$p = 3$$
$$q = 6$$
$$r = 8$$

Hence, $\frac{1}{p} = \frac{1}{3}, \frac{1}{q} = \frac{1}{6}, \frac{1}{r} = \frac{1}{8}$

LCM = 24

$$h = \frac{24}{3} = 8, k = \frac{24}{6} = 4, l = \frac{24}{8} = 3$$

Thus this plane will be represented by the **Miller Indices** (8, 4, 3).

4.11 INTERPLANNER SPACING FOR CUBIC SYSTEM

[Dec. 17]

- The spacing between lattice planes of crystals in which the edges of the unit cell are perpendicular can be found as follows :
- Consider the case of plane ABC of a cubic crystal as shown in Fig. 4.17. Let OA, OB and OC be orthogonal axes. Consider any set of parallel planes defined by the Miller indices (h k l). Consider the plane passing through the origin as the reference plane.
- Let line ON be the normal to the plane passing through the origin, the length of which represents the interplanar spacing d. Let α, β and γ be the angles between co-ordinate axes X, Y, Z and ON respectively. [See Fig. 4.18 (a)].

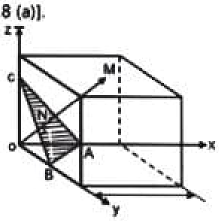


Fig. 4.17

- The intercepts of the plane on the three axes are

$$OA = \frac{a}{h}, \quad OB = \frac{a}{k} \text{ and } OC = \frac{a}{l} \qquad \text{... (4.6)}$$

where 'a' is the cube edge

From Fig. 4.18 (a), we have

$$\cos\alpha = \frac{d}{OA}$$

$$\cos\gamma = \frac{d}{OB} \text{ and } \cos\beta = \frac{d}{OC} \qquad \text{... (4.7)}$$

From Fig. 4.18 (b),

$$ON = [x^2 + y^2 + z^2]^{1/2}$$

$$d = [d^2 (\cos^2\alpha) + d^2 (\cos^2\beta) + d^2 (\cos^2\gamma)]^{1/2}$$

i.e. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ... (4.8)

Substituting the values of cos α, cos β and cos γ from equation (4.7) in equation (4.8), we get

$$\left(\frac{d}{OA}\right)^2 + \left(\frac{d}{OB}\right)^2 + \left(\frac{d}{OC}\right)^2 = 1$$

i.e. $$\left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{b}\right)^2 + \left(\frac{dl}{c}\right)^2 = 1$$

i.e. $$\frac{d^2h^2}{a^2} + \frac{d^2k^2}{b^2} + \frac{d^2 l^2}{c^2} = 1$$

$$\frac{d^2}{a^2}(h^2 + k^2 + l^2) = 1 \qquad \text{(as a = b = c for a cube)}$$

Or $$d_{(hkl)} = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \qquad \text{... (4.9)}$$

This gives the relation between interplanar spacing 'd' and the edge of the cube 'a'.

For general lattice structures,

$$d_{hkl} = \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)^{\frac{1}{2}}$$

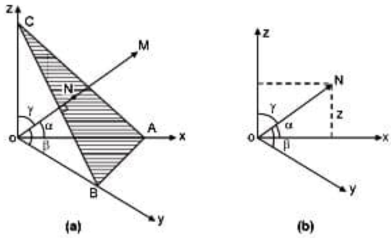


Fig. 4.18

SOLVED PROBLEMS

Problem 4.1 : *Calculate the lattice constant for potassium bromide, given density of potassium bromide is 2700 kg/m^3 and belongs to FCC lattice. Molecular weight of potassium bromide is 119 and Avogadro number is 6.02 × 10^{26} kg mol^{-1}.*

Data : $n = 4,\ M = 119,$

$$N = 6.02 \times 10^{26},\ \rho = 2700$$

Formula : $a = \left(\frac{nM}{N\rho}\right)^{1/3}$

Solution :
$$a = \left(\frac{4 \times 119}{6.02 \times 10^{26} \times 2700}\right)^{1/3}$$
$$= (2.928 \times 10^{-28})^{1/3}$$
$$= \boxed{6.64 \times 10^{-10}\ \text{m}}$$

Problem 4.2 : *NaCl crystal has a lattice constant of 5.643 A°. Given molecular weight of NaCl is 58.45 and Avogadro number is 6 × 10^{26} per kg mole. Find its density.*

Data : $n = 4,\ a = 5.643\ \text{A}°,\ M = 58.45,\ N = 6 \times 10^{26}$ kg mol^{-1}

Formula : $\rho = \frac{nM}{Na^3}$

Solution :
$$\rho = \frac{nM}{Na^3}$$
$$= \frac{4 \times 58.45}{6 \times 10^{26} \times (5.643 \times 10^{-10})^3}$$
$$= \boxed{2168.51\ \text{Kg/m}^3}$$

Problem 4.3 : *Find the Miller indices of a plane making intercepts 3a, 5b, 7c on oblique axes $\overline{a}$, $\overline{b}$, $\overline{c}$.*

Solution :

Taking reciprocals of the intercepts,

$$\frac{1}{p_1} : \frac{1}{p_2} : \frac{1}{p_3} = \frac{1}{3} : \frac{1}{5} : \frac{1}{7}$$

Multiplying by the LCM 105

$$\frac{105}{3} : \frac{105}{5} : \frac{105}{7}$$

Then
$$h = \frac{105}{3} = 35;$$
$$k = \frac{105}{5} = 21;\ L = \frac{105}{7} = 15$$

Miller indices are $\boxed{(35,\ 21,\ 15)}$.

Problem 4.4 : *Calculate the interplanar spacing for (3, 2, 1) plane in a simple cubic lattice where lattice constant is 4.2 × 10^{-10} m.*

Data : $(h\,k\,l) = (3, 2, 1),\ a = 4.2 \times 10^{-10}$ m

Formula : $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

Solution :
$$d_{hkl} = \frac{4.2 \times 10^{-10}}{\sqrt{9 + 4 + 1}} = \frac{4.2}{\sqrt{14}} \times 10^{-10}$$
$$= \boxed{1.1225 \times 10^{-10}\ \text{m}}$$

Problem 4.5 : *A FCC crystal has an atomic radius of 1.246 A°. What are d_{200} d_{230} and d_{111} spacings?*

Solution : For FCC crystal the interatomic distance

$$a = \frac{4r}{\sqrt{2}}$$
$$= 2\sqrt{2}\ r$$

Given : Atomic radius r = 1.246 A°

$$a = 2\sqrt{2} \times 1.246$$
$$a = 3.524\ \text{A}°$$

For a crystal, $d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$

(i) $d_{200} = \frac{3.524}{\sqrt{2^2 + 0 + 0}} = \frac{3.524}{2}$

∴ $d_{200} = 1.762\ \text{A}°$

(ii) $d_{220} = \frac{3.524}{\sqrt{4 + 4 + 0}} = 1.245\ \text{A}°$

(iii) $d_{111} = \frac{3.524}{\sqrt{1 + 1 + 1}}$
$$= \boxed{2.034\ \text{A}°}$$

Problem 4.6 : *Find out the number of atoms per square millimeter on a plane (100) of lead whose interatomic distance is 3.499 A°. Lead has face-centred cubic structure.*

Solution :

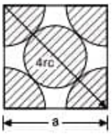


Fig. 4.19

For Fig. 4.19 represents (100) plane. The shaded portion shows the portion of the atom.

For FCC structure,

$$r = \frac{\sqrt{2}\,a}{4}$$

$$\therefore \quad a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}\ r$$

Interatomic distance = 3.499

$$\therefore \quad 2r = 3.499$$

Now, $a = \sqrt{2}\ (3.499) = 4.95\ A^\circ$

area of plane $= a^2$

where, a is side of given plane

$$= (4.95 \times 10^{-7})^2\ mm^2$$

Number of atoms in the plane = 2

$$\text{Atoms/mm}^2 = \frac{2 \times 1}{(4.95 \times 10^{-7})^2}$$

$$= \boxed{8.2 \times 10^{12}\ \text{atoms/mm}^2}$$

***Problem 4.7 :** Calculate the number of atoms per unit cell of metal having a lattice parameter of 2.9 A° and density 7.87 gram/cc. Atomic weight of the metal is 55.85 and Avogadro constant is 6.023×10^{23}.*

Solution : Density of crystal $\rho = \frac{nM}{a^3 N_A}$

Given : $a = 2.9\ A^\circ = 2.9 \times 10^{-8}$ cm, M = 55.85,

$N_A = 6.023 \times 10^{23}$, $\rho = 7.87$ gm/cc.

$$n = \frac{\rho\, a^3 N_A}{M}$$

$$= \frac{7.87 \times (2.9 \times 10^{-8})^3 \times 6.023 \times 10^{23}}{55.85}$$

$$n = \boxed{2\ \text{atoms}}$$

Hence, unit cell may be body centred cubic.

***Problem 4.8 :** Sketch (112) plane in simple cubic cell.*

Solution : A plane whose Miller indices are (112) has the following intercepts on the three axes.

$$\frac{1}{1}, \frac{1}{1}, \frac{1}{2}$$

This plane is shown in Fig. 4.20.

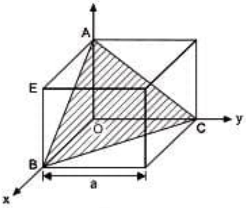


Fig. 4.20

Here,

$$OB = OC = a$$

and $$OA = \frac{a}{2}$$

***Problem 4.9 :** Sketch (110), (010), (001), (200), ($\bar{1}$00) and (112) planes in simple cubic cell.*

Solution : (i) A plane whose Miller indices are (100) has intercepts $\frac{1}{1}, \frac{1}{0}, \frac{1}{0}$

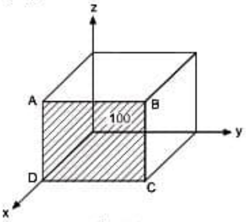


Fig. 4.21

(ii) A plane whose Miller indices are (101) has intercepts

$$\frac{1}{1}, \frac{1}{0}, \frac{1}{1}$$

i.e. $1, \infty, 1$

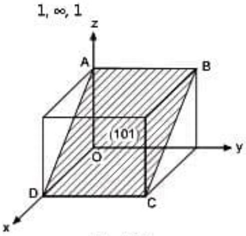


Fig. 4.22

(iii) A plane whose Miller indices are (110) has intercepts

$$\frac{1}{1}, \frac{1}{1}, \frac{1}{0}$$

i.e. $1, 1, \infty$

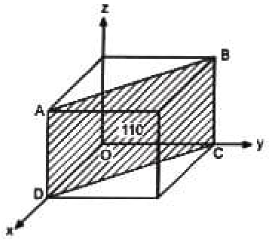


Fig. 4.23

(iv) A plane whose Miller indices are (010) has intercepts.

$$\frac{1}{0}, \frac{1}{1}, \frac{1}{0}$$

i.e. $\infty, 1, \infty$

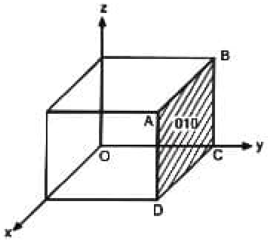


Fig. 4.24

(v) A plane whose Miller indices are (001) has intercepts

$$\frac{1}{0}, \frac{1}{0}, \frac{1}{1}$$

i.e. $\infty, \infty, 1$

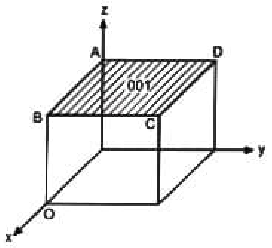


Fig. 4.25

(iv) A plane whose Miller indices are (2, 0, 0) has intercepts

$$\frac{1}{2}, \frac{1}{0}, \frac{1}{0}$$

i.e. $\frac{1}{2}, \infty, \infty$

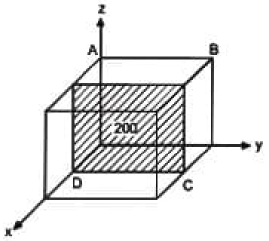


Fig. 4.26

(vii) A plane whose Miller indices are $(\bar{1},0,0)$ has intercepts

$$\frac{-1}{1}, \frac{1}{0}, \frac{1}{0}$$

i.e. $-1, \infty, \infty$

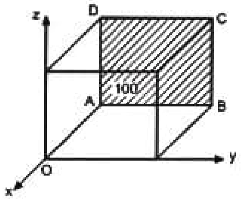


Fig. 4.27

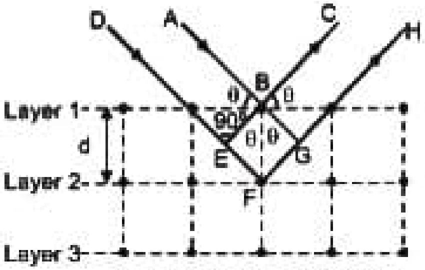
4.12 X-RAY DIFFRACTION

- Because of the short wavelength of X rays ($\approx 10^{-10}$ m), the ordinary ruled grating with approximately 6×10^5 Lines per meter, used with ordinary light, cannot produce any appreciable diffraction.
- But a crystal in which, the atoms arranged in a regular pattern correspond to the grating lines and the atomic spacing (of the order of the X-ray wavelength), forms the grating element exhibits diffraction. The crystal differs from an optical grating in that the former is a **Three Dimensional Space Grating** while the latter is a two dimensional plane grating.
- According to Laue if a narrow pencil of X-rays was made to pass through a thin crystal, a diffraction pattern can be obtained. This symmetrical pattern of spots is known as Laue pattern and it proves that X-rays are electromagnetic waves.

- Prof. W. L. Bragg gave a simple interpretation of the diffraction pattern obtained when a beam of monochromatic X-rays was made to pass through a crystal.
- The atoms in a crystal are arranged in a regular three dimensional manner in such a way that there are sets of parallel layers rich in atoms in different planes, each plane having its own spacing between layers, some planes richer in atoms than others.
- When a beam of X-rays falls on the crystal, each atom acts as a scattering centre and emits secondary wavelets whose envelope gives rise to the reflected wavefront (by Huygen's theory). Hence scattering of X-rays can be looked upon as Bragg reflections from planes called the **Bragg Planes**.
- At certain glancing angles, reflections from these set of parallel planes are in phase (to give maximum intensity). At other angles, reflections from different planes are out of phase (to give minimum or zero intensity).

4.13.1 Bragg's Law [Dec. 18]

- Consider a crystal consisting of a set of parallel planes separated by a distance d.
- Let a narrow monochromatic beam of X-rays of wavelength λ be incident on this plane at a glancing angle θ.
- Consider two parallel rays ABC and DFH which are reflected by two atoms B and F in adjacent layers, F being vertically below B.
- Ray DFH has a longer path than ray ABC and the path difference between them is given by (EF + FG) where E and G are the feet of the perpendiculars drawn from B on DF and DH respectively.
- This path difference will decide whether the two reflected rays will be in phase or out of phase.
- When the path difference is equal to nλ (n-an integer), the reflected rays will reinforce each other to produce an intense beam.



Reflection of X-rays from a crystal plane

Fig. 4.28

$$EF + FG = n\lambda \qquad \text{... (4.10)}$$

From Fig. 4.28,

$$EF = FG = d \sin \theta \qquad \text{... (4.11)}$$

Substituting (4.11) in (4.10),

$$2 d \sin \theta = n\lambda$$

- This relation is known as **Bragg's Law**. It indicates that, for given values of n, λ and d there is reflection only in a particular direction defined by θ.
- Destructive interference occurs in other directions.
- By making n equal to 1, 2, 3 etc. successively, a series of values are obtained for θ for which sharp reflections occur.
- These are called as first order (n = 1), second order (n = 2), etc. maximum. As the order of the spectrum increases, the intensity decreases.

4.13.2 Bragg's X-Ray Spectrometer

- It consists of three parts : (1) a source of X-rays suitably collimated, (2) a circular table graduated and provided with a vernier to hold the crystal and (3) a deflecting device.
- X-rays from an X-ray tube are collimated into a fine beam by two narrow slits S_1 and S_2.
- The beam is made to be incident at a glancing angle θ on the face of a crystal C (of calcite, rock salt, mica, NaCl etc.), which is mounted on a circular table T. This table can rotate about a vertical axis and its position can be read by the graduated scale and vernier V.
- Rotating about the same axis is an arm R carrying an ionization chamber I, provided with a vernier to fix its position.
- The intensity of the X-ray beam (i.e. degree of ionisation in the chamber) that enters the chamber is measured by an electrometer E.

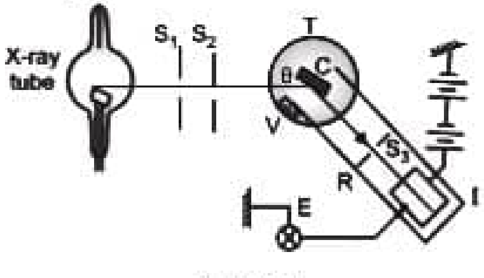
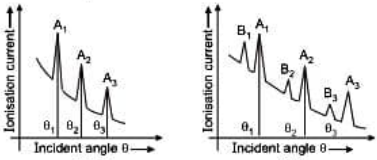


Fig. 4.29

- The turn table T and arm R are so linked that when the T, and hence the crystal, rotates through an angle θ, the R and hence the ionization chamber 'I' turns through 2θ.
- Therefore, whatever be the incident angle of the beam at the crystal surface, it is always reflected into I. If the reflected beam is to be recorded on a photographic film, I can be replaced by a camera.
- The ionization current is measured for different values of θ and a plot is obtained.



(a) (b)

Fig. 4.30

- From the graph, the glancing angles θ_1, θ_2, θ_3 for first, second and third order reflections are measured. Knowing the values of d and n, wavelength λ of X-rays can be calculated using Bragg's equation i.e. $2d \sin\theta = n\lambda$.
- Fig. 4.30 (a) is the spectrum when the X-ray beam is monochromatic while Fig. 4.30 (b) represents the case when the X-ray beam consists of two wavelengths λ_1 and λ_2.
- In this way, Bragg's spectrometer can be used to find the wavelength of X-rays. Conversely d can be computed if λ of X-rays is known from some other experiment.

Problem 4.10 : *Using Bragg's X-ray spectrometer, the glancing angle for first order spectrum was observed to be 8°. Calculate the wavelength of X-rays if $d = 2.82 \times 10^{-10}$ m.*

Data : $\theta = 8°$,

$d = 2.82 \times 10^{-10}$ m, n = 1

Formula : $2d \sin\theta = n\lambda$

Solution : $\lambda = \frac{2d}{n} \sin\theta$

$$= \frac{2 \times 2.82 \times 10^{-10} \sin 8°}{1}$$

$$= 2 \times 2.82 \times 10^{-10} \times 0.1392 \text{ m}$$

$$= \boxed{0.7857 \text{ A}°}$$

Problem 4.11 : *X-rays of wavelength 1.6 A° are diffracted by a Bragg crystal spectrometer at an angle 14.2° in the first order. What is the spacing of atomic layers in the crystal ?*

Data : $\lambda = 1.6 \text{ A}° = 1.6 \times 10^{-10}$ m, $\theta = 14.2°$, n = 1

Formula : $2d \sin\theta = n\lambda$

i. e. $d = n \frac{\lambda}{2 \sin\theta}$

Solution : $= 1 \times 1.6 \times \frac{10^{-10}}{2 \times \sin 14.5°}$

$$= \frac{1.6 \times 10^{-10}}{2 \times 0.2454} = \boxed{3.26 \text{ A}°}$$

Problem 4.12 : *Calculate the longest wavelength that can be analysed by a crystal with an interplanar spacing of 3 A° in first order.*

Data : d = 3 A°, n = 1, $\sin\theta = 1$ (maximum)

Formula : $\lambda = \frac{2d}{n} \sin\theta$

Solution : $\lambda = 2 \times 3\text{A}°$

$$= \boxed{6 \text{ A}°}$$

Problem 4.13 : *The unit cell dimension 'a' of NaCl lattice is 5.63 A°. If an X-ray beam of $\lambda = 1.1$ A° falls on a family of planes with a separation of $\frac{a}{\sqrt{5}}$, how many orders of diffraction are observable ?*

Data : $\lambda = 1.1$ A°, a = 5.63 A°, $d = \frac{a}{\sqrt{5}}$

Formula : $n = \frac{2d \sin\theta}{\lambda}$

Solution : For maximum observable orders, θ = 90°

$$\therefore \quad n = \frac{2d}{\lambda} = \frac{2 \times 5.63}{\sqrt{5}} \times \frac{1}{1.1}$$

$$= \boxed{4.585 < 5}$$

Four orders of diffraction are observable.

Problem 4.14 : *When a nickel target is used in a X-ray tube, the two shortest wavelengths emitted are found with a Bragg's crystal spectrograph to be diffracted at an angle of 15.1° and 17.1° respectively. Find their wavelength assuming crystal spacing of 2.81 A°.*

Data : d = 2.81 A°, $\theta_1 = 15.1°$, $\theta_2 = 17.1°$, n = 1

Formula : $\lambda = \frac{2d \sin\theta}{n}$

Solution :

$$\lambda_1 = \frac{2d \sin \theta}{n} = 2 \times 1.81 \times \sin 15.1^\circ$$

$$= 1.464 \text{ A}^\circ$$

$$\lambda_2 = \frac{2d \sin \theta}{n} = 2 \times 1.81 \times \sin 17.1^\circ$$

$$= \boxed{1.652 \text{ A}^\circ}$$

4.13.3 Crystal Structure using X-Rays Diffraction

- Atoms of different elements in a crystal have definite orderly arrangement in the space. When X-rays are incident on the crystal, they are scattered by each atom of the crystal. Definite arrangement of atoms represent sets of parallel planes which are rich in these atoms.
- Combined effect of scattering of X-rays by these sets of planes can be considered as reflection of X-rays by these planes.
- The scattering caused by the atoms is called **Bragg's Scattering** or **Bragg's Reflection**. The planes which cause these reflections are called **Bragg's Planes**.
- Due to the presence of sets of parallel planes rich in certain types of atoms, the crystal acts like diffraction grating for X-rays.
- X-ray reflections from these sets of parallel planes are in phase with each other for certain glancing angle. Hence, different reflected X-rays combine together to give a very strong effect. As a result the ionisation current produced by these is very large. If the angles of incidence are other than the above angle, X-ray reflections are antiphase with each other. So the resultant reflection is zero or extremely weak.

4.14LINE AND CONTINUOUS SPECTRUM OF X-RAY

- The X-rays are produced when high velocity electrons strike the target material of high atomic number, such as tungsten or molybdenum. About 99 % of electrons striking solid matter targets are wasted in heating the target and increasing the kinetic energy of the particles hit. The remaining 1 % collisions produce two types of X-rays, the characteristic X-rays and continuous X-rays, by loosing their energy in the following two ways.
  1. Some of the incident electrons transfer their energy to the target atoms. The target atoms retain this energy temporarily as excitation energy and then emit it shortly as X-rays which are characteristic of the target material. So they are called **Characteristic X-Rays**.
  2. Other electrons are stopped by the target and their energy is directly converted into X-rays. There is continuous loss in kinetic energy of incident electrons and X-rays are emitted continuously. So they are called **Continuous X-Rays**.
- The basic requirements of X-ray production are : (i) A source of electrons (ii) Effective means of accelerating electrons and (iii) A target of suitable material of high atomic number.
- There are two types of X-ray tubes. (i) The gas filled or Roentgen X-ray tube and (ii) The Coolidge hot cathode X-ray tube.
- In Coolidge X-ray tube, it is possible to achieve separate control of the intensity and quality of X-rays independent of each other. So we describe it here.

4.14.1 Coolidge Hot-Cathode X-Ray Tube

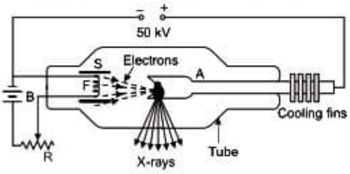


Fig. 4.31 : Coolidge tube

- This tube is widely used for commercial and medical purposes. The essential elements of a modern Coolidge X-ray tube are shown in Fig. 4.31.
- A tungsten filamentary cathode F is heated by a low voltage battery and electrons are produced thermionically. These electrons are focused on the target T by a cylindrical shield S which surrounds F. The shield S is maintained at a negative potential. The electrons are accelerated to very high speeds by the d.c. voltage of 50 kV to 100 kV applied between the cathode the F and the anode.
- This high d.c. voltage is obtained from a step up transformer whose output is converted into d.c. voltage by a full wave rectifier.
- The target T consists of a copper block in which a piece of tungsten or molybdenum is fitted. The face of the copper anode is sloped at 45° to the electron beam.
- The copper block conducts heat efficiently to the external cooling fins. The anode should be of a metal of high melting point, so that, it will not melt under the bombardment of electrons.

- To produce hard X-rays and for their abundant yield, the anode should be of a metal of high atomic number.
- When the accelerated electrons strike the target and are stopped by it, they give up their kinetic energy and thereby produce X-rays.
- The intensity of X-rays depends on the number of electrons striking the target. This number depends on the temperature of the filament and so on the filament current. So, by controlling the filament current by a rheostat R, the thermionic emission and hence intensity of X-rays can be controlled.
- The quality of X-rays is measured in terms of their penetrating power. It depends on the p.d. between the cathode and the anode.
- Greater the accelerating voltage, higher is the speed of the striking electrons and consequently, more penetrating are the X-rays produced.
- High penetrating X-rays are called hard X-rays and low penetrating X-rays are called soft X-rays. Thus the quality of X-rays in Coolidge tube can be controlled by varying the p.d. between the cathode and the anode.

4.14.2 Continuous X-Rays

[Dec. 17, May 19]

- When fast moving electrons are obstructed by solid metal targets, X-rays are produced. Some of the high velocity electrons penetrate deep into the interior of the atoms of the target material and are attracted by their positively charged nuclei. As an electron passes close to the nucleus, it is deflected from its path as shown in Fig. 4.32.
- The electron experiences deceleration during its deflection in the strong field of the nucleus. The energy lost during its retardation is given out in the form of X-rays of continuously varying wavelength.
- These X-rays produce a continuous spectrum having a sharply defined short wavelength limit λ_{min} (or high frequency limit υ_{max}) which corresponds to the maximum energy of the incident electron.
- As shown in Fig. 4.32, if the striking electron of mass m has its velocity reduced from v to v' due to deflection or due to collision, then loss of its energy is $\left(\frac{1}{2}\ mv^2 - \frac{1}{2}\ mv'^2\right)$ and this must be equal to the energy hυ of the emitted X-ray photon,

$$\therefore \quad h\upsilon = \frac{1}{2}\ m\ (v^2 - v'^2)$$

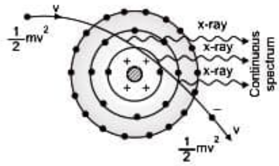


Fig. 4.32

- When the electron is completely stopped, i.e. when v' = 0, X-rays of maximum frequency are emitted. In that case,

$$h\upsilon_{max} = \frac{1}{2}\ mv^2 \qquad \dots (4.12)$$

- If kinetic energy is imparted to the electron by accelerating it through a p.d. of V volts, then

$$eV = \frac{1}{2}\ mv^2 \qquad \dots (4.13)$$

From equations (4.12) and (4.13)

$$h\upsilon_{max} = eV$$

$$\text{i.e.} \quad h\frac{c}{\lambda_{min}} = eV \qquad (\because \lambda_{min}\ \upsilon_{max} = c)$$

$$\therefore \quad \lambda_{min} = \frac{hc}{eV} \qquad \dots (4.14)$$

where c is the velocity of light.

- Equation (4.14) gives the short-wavelength limit of the continuous X-ray spectrum.

Substituting for the constants

$c = 3 \times 10^8$ m/s, $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J-s,

we get,

$$\lambda_{min} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}\ V} \text{ metres}$$

$$= \frac{1.24 \times 10^{-6}}{V} \text{ m}$$

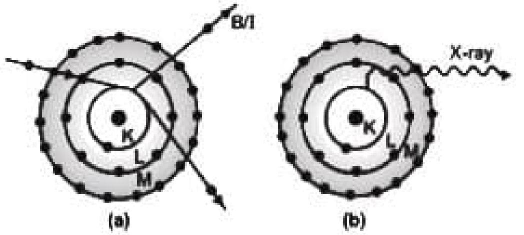
$$\therefore \quad \lambda_{min} = \frac{12400}{V}\ A^{\circ} \qquad \dots (4.15)$$

- Such radiations are called **Braking Radiations**, because they are due to braking or slowing down of high velocity electrons in the positive field of a nucleus. These X-rays consist of a series of uninterrupted wavelengths having a sharply defined short wavelength limits λ_{min} and they form the continuous spectrum.

- They are determined by the p.d. between the cathode and the anode of the X-ray tube and are independent of the nature of the target material.

4.14.3 Characteristic of X-Rays

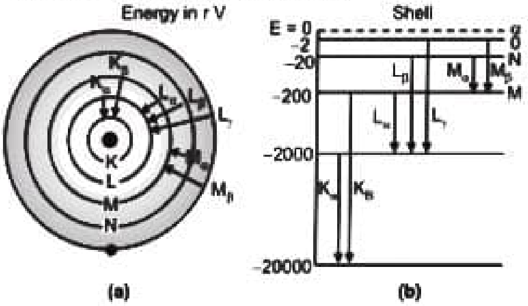
- When fast moving electrons are suddenly stopped by solid metal targets, X-rays are produced. Some of the high velocity electrons knock out the tightly bound electrons in the innermost shells (like K, L, M... etc. shells) of the atoms, while penetrating the interior of the atoms of the target material.
- When electrons from outer orbits jump to fill up the vacancy so produced, the energy difference is given out in the form of X-rays of definite wavelength.
- These wavelengths form the line spectrum which is characteristic of the material of the target. So it is called **Characteristic X-Ray** spectrum.



(a) (b)

Fig. 4.33

- Fig. 4.33 (a) shows the case when the high velocity incident electrons knock out one electron from the K-shell of the atom. As shown in Fig. 4.33 (b), this K-shell vacancy is filled up by an electron from a nearby L-shell.
- During this jump an X-ray photon of frequency υ, such that $E_K - E_L = h\upsilon$ is emitted, where E_K is the energy required to dislodge a K-shell electron and E_L is the energy required to dislodge an L-shell electron.
- As this energy difference $(E_K - E_L)$ is comparatively very large, the emitted X-rays have very large energy and hence they are highly penetrating.



(a) (b)

Fig. 4.34

- If, however, the K-shell vacancy is filled up by an electron jumping from the M-shell, the emitted X-rays would be still more energetic and would possess still higher frequency because $(E_K - E_M) > (E_K - E_L)$. Such X-rays arising from millions of atoms produce the K-lines of the characteristic spectrum as shown in Fig. 4.34 (a). Usually, K_α and K_β lines of this series are detected although there are many more.

  Similarly, when the incident electron has somewhat lesser energy, it dislodges an L-shell electron. This L-shell vacancy is filled up by an electron jumping from M-shell to other outer shells, and X-rays of frequency lower than that of K-series are produced. This gives the L_α, L_β, and L_γ lines of the L-series of the characteristic spectrum, as shown in Fig. 4.34 (a) and (b).
- Spectral lines of M-series are produced when incident electron knocks out an M-shell electron and this M-shell vacancy is filled up by electrons jumping from outer shells. The characteristic X-ray lines are shown in energy level diagram in Fig. 4.34 (a) and (b).
- These K-series, L-series and M-series constitute the line spectra of the X-rays, which are characteristic of the material of the target used in the X-ray tube.
- So, the X-rays produced by an X-ray tube consist of two parts.
  1. One part consists of a series of uninterrupted wavelengths having a short cut off λ_{min}. This constitutes the continuous spectrum.
  2. The other part consists of a number of distinct and discrete wavelengths. They constitute the line or the characteristic X-ray spectrum.
- The characteristic spectrum is superposed on the continuous spectrum. X-ray spectrum of molybdenum is shown in Fig. 4.35.

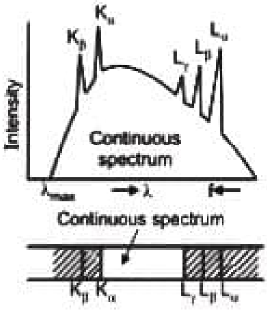


Fig. 4.35

4.15 MOSELEY'S LAW [May 18]

- A systematic study of characteristic X-ray spectra of different metallic elements was carried out by Moseley in 1913 - 14 by using them as targets in the X-ray tube. He used Bragg's spectrometer for this study.
- His observations are :
  - ➢ The characteristic X-ray spectra of different elements are similar, as characteristic spectrum of any element consists of K-series, L-series and M-series.
  - ➢ The frequency of lines (in every series) produced from an element of higher atomic number is greater than that produced by an element of lower atomic number. This is so because the binding energy of electrons increases as we go from one element to another of higher atomic number, i.e. larger amount of energy is required to liberate an electron from K-shells, L-shells and M-shells of atoms of an element of higher atomic number.
- Thus, say for K_α line, the higher the atomic number of the target material, higher is the frequency of the K_α line. Mathematically,

$$\upsilon \propto (Z-a)^2$$

or $$\upsilon = b\,(Z-a)^2 \qquad \dots (4.16)$$

where υ = frequency of characteristic radiation.

b = constant, which is different for different series.

a = constant, known as screening constant and is different for different series.

Z = Atomic number of the target material.

- Equation (4.16) is called Moseley's law. It may be stated as, "The frequency of a spectral line in characteristic X-ray spectrum varies directly as the square of the atomic number of the element emitting it".
- Moseley's diagram for K_α and K_β lines, obtained by plotting $\sqrt{\upsilon}$ versus Z of different elements is shown in Fig. 4.36.

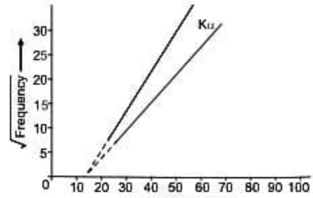


Fig. 4.36

- For K_α line, it was found that $b = \frac{3}{4}$ R, where R is Rydberg constant, and a = 1.

Hence for K_α line,

$$\upsilon_{K_\alpha} = \frac{3}{4} R\,(Z-1)^2 \qquad \dots (4.17)$$

Importance of Moseley's Law :

- As per this law, the atomic number of an element determines its physical and chemical properties, and not its atomic weight. So the basis of periodic table is now taken as to arrange elements according to their increasing atomic number and not as their increasing atomic weights as was done earlier.
- Mandelev's basis of the periodic table was to arrange elements in ascending atomic weights. So, Potassium $_{19}K^{39}$ was placed before Argon $_{18}A^{40}$ and Nickel and $_{28}Ni^{58.7}$ was placed before Cobalt $_{27}Co^{58.9}$. However, Mandelev observed that to maintain the periodicity of chemical and physical properties their orders should be reversed. This difficulty was removed by Moseley's law, because as per their atomic numbers, their order should be just opposite.
- Moseley's law also led to the discovery of some new elements like Hafnium (Z = 72), Promethenium (Z = 61), Technitium (Z = 43), Rhenium (Z = 75) etc.

Problem 4.15 : *An X-ray tube operating at 20 kV emits a continuous spectrum with shortest wavelength limit of 0.62 A°. Calculate Planck's constant.*

Solution : Short wavelength limit

$$\frac{hc}{eV} = \lambda_{min}$$

$$\therefore \quad h = \frac{eV\,\lambda_{min}}{C}$$

$$= \frac{1.6 \times 10^{-19} \times 2 \times 10^{4} \times 0.62 \times 10^{-10}}{3 \times 10^{8}}$$

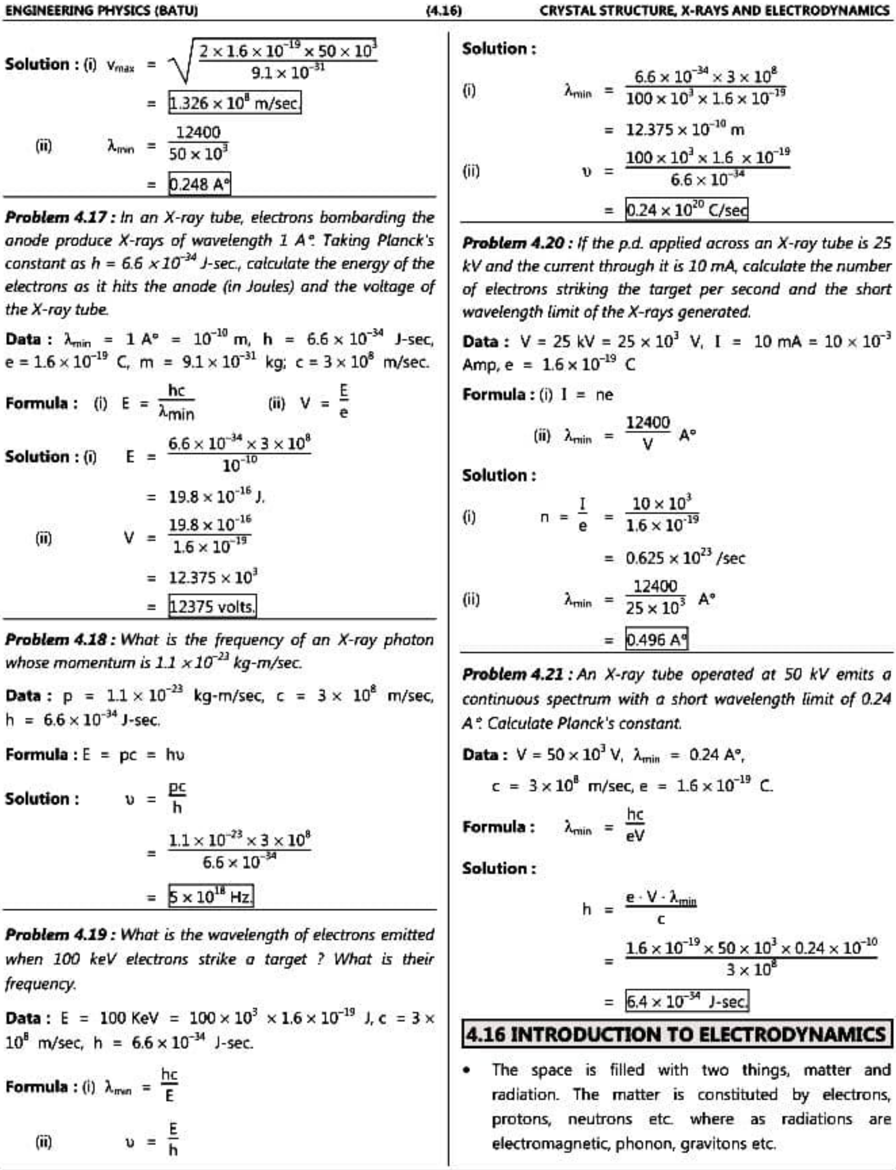
$$\therefore \quad h = \boxed{6.72 \times 10^{-34}\ Js}$$

Problem 4.16 : *A Coolidge tube operates at 50 kV. Find (i) the maximum velocity of the electrons striking the anticathode (ii) minimum wavelength of X-rays generated. Given $e = 1.6 \times 10^{-19}$C, $m = 9.1 \times 10^{-31}$ kg.*

Data : $V = 50\ kV = 50 \times 10^3\ V$, $e = 1.6 \times 10^{-19}\ C$, $m = 9.1 \times 10^{-31}\ kg$

Formula : (i) $v_{max} = \sqrt{\frac{2\,eV}{m}}$

(ii) $\lambda_{min} = \frac{12400}{V}\ A°$



Solution : (i) $v_{max} = \sqrt{\dfrac{2 \times 1.6 \times 10^{-19} \times 50 \times 10^{3}}{9.1 \times 10^{-31}}}$

$= \boxed{1.326 \times 10^{8} \text{ m/sec}}$

(ii) $\lambda_{min} = \dfrac{12400}{50 \times 10^{3}}$

$= \boxed{0.248 \text{ A}^\circ}$

Problem 4.17 : *In an X-ray tube, electrons bombarding the anode produce X-rays of wavelength 1 A°. Taking Planck's constant as $h = 6.6 \times 10^{-34}$ J-sec., calculate the energy of the electrons as it hits the anode (in Joules) and the voltage of the X-ray tube.*

Data : $\lambda_{min} = 1 \text{ A}^\circ = 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg; $c = 3 \times 10^{8}$ m/sec.

Formula : (i) $E = \dfrac{hc}{\lambda_{min}}$ (ii) $V = \dfrac{E}{e}$

Solution : (i) $E = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10}}$

$= 19.8 \times 10^{-16}$ J.

(ii) $V = \dfrac{19.8 \times 10^{-16}}{1.6 \times 10^{-19}}$

$= 12.375 \times 10^{3}$

$= \boxed{12375 \text{ volts.}}$

Problem 4.18 : *What is the frequency of an X-ray photon whose momentum is 1.1×10^{-23} kg-m/sec.*

Data : $p = 1.1 \times 10^{-23}$ kg-m/sec, $c = 3 \times 10^{8}$ m/sec, $h = 6.6 \times 10^{-34}$ J-sec.

Formula : $E = pc = h\upsilon$

Solution : $\upsilon = \dfrac{pc}{h}$

$= \dfrac{1.1 \times 10^{-23} \times 3 \times 10^{8}}{6.6 \times 10^{-34}}$

$= \boxed{5 \times 10^{18} \text{ Hz.}}$

Problem 4.19 : *What is the wavelength of electrons emitted when 100 keV electrons strike a target ? What is their frequency.*

Data : $E = 100$ KeV $= 100 \times 10^{3} \times 1.6 \times 10^{-19}$ J, $c = 3 \times 10^{8}$ m/sec, $h = 6.6 \times 10^{-34}$ J-sec.

Formula : (i) $\lambda_{min} = \dfrac{hc}{E}$

(ii) $\upsilon = \dfrac{E}{h}$

Solution :

(i) $\lambda_{min} = \dfrac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{100 \times 10^{3} \times 1.6 \times 10^{-19}}$

$= 12.375 \times 10^{-10}$ m

(ii) $\upsilon = \dfrac{100 \times 10^{3} \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}}$

$= \boxed{0.24 \times 10^{20} \text{ C/sec}}$

Problem 4.20 : *If the p.d. applied across an X-ray tube is 25 kV and the current through it is 10 mA, calculate the number of electrons striking the target per second and the short wavelength limit of the X-rays generated.*

Data : $V = 25 \text{ kV} = 25 \times 10^{3}$ V, $I = 10 \text{ mA} = 10 \times 10^{-3}$ Amp, $e = 1.6 \times 10^{-19}$ C

Formula : (i) $I = ne$

(ii) $\lambda_{min} = \dfrac{12400}{V}$ A°

Solution :

(i) $n = \dfrac{I}{e} = \dfrac{10 \times 10^{3}}{1.6 \times 10^{-19}}$

$= 0.625 \times 10^{23}$ /sec

(ii) $\lambda_{min} = \dfrac{12400}{25 \times 10^{3}}$ A°

$= \boxed{0.496 \text{ A}^\circ}$

Problem 4.21 : *An X-ray tube operated at 50 kV emits a continuous spectrum with a short wavelength limit of 0.24 A°. Calculate Planck's constant.*

Data : $V = 50 \times 10^{3}$ V, $\lambda_{min} = 0.24$ A°,

$c = 3 \times 10^{8}$ m/sec, $e = 1.6 \times 10^{-19}$ C.

Formula : $\lambda_{min} = \dfrac{hc}{eV}$

Solution :

$h = \dfrac{e \cdot V \cdot \lambda_{min}}{c}$

$= \dfrac{1.6 \times 10^{-19} \times 50 \times 10^{3} \times 0.24 \times 10^{-10}}{3 \times 10^{8}}$

$= \boxed{6.4 \times 10^{-34} \text{ J-sec.}}$

4.16 INTRODUCTION TO ELECTRODYNAMICS

- The space is filled with two things, matter and radiation. The matter is constituted by electrons, protons, neutrons etc. where as radiations are electromagnetic, phonon, gravitons etc.

- Ordinary matter is made up of atoms which have positively charged nuclei and negatively charged electrons orbiting around them.
- The charge of an atom is quantized in terms of the electronic charge –e whose value is 1.6×10^{-19} coulomb. When two charges are separated a distance, would experience a force due to the electric field produced by them.
- On the other hand the motion of charges generate current and hence magnetic field. When these fields are time varying they produce electromagnetic waves are coupled with each other by Maxwell's equation.
- With the help of Maxwell's equations, we can derive wave equation, based on which the propagation of electromagnetic waves can be investigated in different media.

4.17 INTRODUCTION OF MAXWELL'S EQUATION

[Dec. 18]

- When the charges are in motion, the electric and magnetic fields are associated with them which will change in both the space and time. Thus, the electric and magnetic fields are interrelated with each other. This phenomenon is called **Electromagnetism** which are mathematically explained by **Maxwell's Equations.** We can write Maxwell's equations in differential or integral form.

4.17.1 Differential form of Maxwell's Equation

[May 19]

In differential form, the Maxwell's equations are as given below (S. I. Units)

$$\nabla \cdot \vec{D} = \rho \text{ or } \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{Gauss's law} \; \ldots (4.18)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{Monopoles do not exist} \; \ldots (4.19)$$

$$\nabla \cdot \vec{E} = \frac{-\partial \vec{B}}{\partial t} \quad \text{Faraday's law} \; \ldots (4.20)$$

$$\nabla \cdot \vec{H} = \vec{j} \quad \text{Ampere's circuital law} \; \ldots (4.21)$$

These are the Maxwell's four equations which govern electromagnetism.

- **Displacement Current :** [Dec. 18]
  - The first three of these are general equations and are valid for static as well as dynamic fields.
  - But the fourth equation is derived for steady state, for time-varying fields take the divergence on both sides.

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{j} = 0 \quad \ldots (4.22)$$

The above equation is incompatible with the principle of conservation of charge in the equation of continuity.

i.e. $\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0 \quad \ldots (4.23)$

This is due to incomplete definition of current density. Thus the total current density can be written as

$$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t} = \frac{-\partial}{\partial t} (\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{j} = \nabla \cdot \left(-\frac{\partial \rho}{\partial t}\right)$$

$$\therefore \nabla \cdot \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right) = 0 \quad \ldots (4.24)$$

Maxwell replaced $\vec{j}$ in Ampere's law by $\vec{j} + \frac{\partial D}{\partial t}$.

Therefore, the Ampere's law takes the from

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \quad \ldots (4.25)$$

- The above equation is valid for steady state phenomenon and is also compatible with the equation of continuity for time dependent fields.
- The term $\vec{j}$ is called **Conduction Current Density** and $\frac{\partial D}{\partial t}$ is called **Displacement Current Density.**

4.17.2 Integral Form of Maxwell's Equation

[Dec. 18]

The Maxwell's equations in integral form are

(i) $\oint_s \vec{D} \cdot d\vec{S} = q \quad \ldots (4.26)$

(ii) $\oint_s \vec{B} \cdot d\vec{S} = 0 \quad \ldots (4.27)$

(iii) $\oint \vec{E} \cdot d\vec{l} = \frac{-\partial}{\partial t} \int_s \vec{B} \cdot d\vec{S} \quad \ldots (4.28)$

(iv) $\oint \vec{H} \cdot d\vec{l} = \int_s \left[\vec{j} + \frac{\partial \vec{D}}{\partial t}\right] d\vec{S} \quad \ldots (4.29)$

4.18 ELECTROMAGNETIC WAVE IN FREE SPACE

[Dec. 17]

The Maxwell's equations for free space are given by,

$$\nabla \times \vec{E} = 0 \quad \ldots (4.30)$$

$$\nabla \times \vec{H} = 0 \quad \ldots (4.31)$$

$$\nabla \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} \quad \text{... (4.32)}$$

and $$\nabla \times \vec{E} = \varepsilon_o \frac{\partial \vec{E}}{\partial t} \quad \text{... (4.33)}$$

Taking Curl of equation (4.32)

$$\nabla \times (\nabla \times \vec{E}) = -\mu_o \frac{\partial}{\partial t} (\nabla \cdot \vec{H})$$

or $$\nabla \times (\nabla \times \vec{E}) - \nabla^2 \vec{E} = -\mu_o \frac{\partial}{\partial t} \left[\varepsilon_o \frac{\partial \vec{E}}{\partial t} \right]$$

But $$\nabla \cdot \vec{E} = 0$$

$$\therefore \quad \nabla^2 \vec{E} = -\mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{... (4.34)}$$

This is the wave equation governing the field $\vec{E}$.

We can write this as

$$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{... (4.35)}$$

where, $$v = \frac{1}{\sqrt{\mu_o \varepsilon_o}} \quad \text{... (4.36)}$$

Similarly the curl of equation (4.33) gives the wave equation for the field $\vec{H}$.

i.e. $$\nabla^2 \vec{H} - \mu_o \varepsilon_o \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \text{... (4.37)}$$

if $$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{... (4.38)}$$

then $$\nabla^2 \vec{H} - \frac{1}{v^2} \frac{\partial^2 \vec{H}}{\partial t^2} = 0 \quad \text{... (4.39)}$$

The solution of equations (4.35) and (4.36) is written as

$$\vec{E}(\vec{r}, t) = \vec{E}_o e^{i(\vec{K} \cdot \vec{r} - \omega t)} \quad \text{... (4.40)}$$

and $$\vec{H}(\vec{r}, t) = \vec{H}_o e^{i(\vec{K} \cdot \vec{r} - \omega t)} \quad \text{... (4.41)}$$

where, ω is the angular frequency of variation of the fields $\vec{E}$ and $\vec{H}$.

$\vec{K}$ is the wave vector which tells the direction of propagation of the wave.

The ratio ω/K gives the phase velocity of the wave which is equal to the speed of light c in the free space or vacuum.

In free space (vacuum)

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ f/m}$$

From equ. (4.38),

$$\therefore \quad C = 2.99 \times 10^8 \text{ m/s},$$

which is the velocity of light.

SUMMARY

- **Space Lattice :** It is defined as a regular 3-dimensional periodically repetitive arrangement of points in space, which is infinite in extent.
- **Basis :** It is a group of atoms or molecules.
- **Crystal Structure :** It is formed by the addition of basis to every lattice point of space lattice.
- **Unit Cell :** The parallelopiped formed by using premitive reactors ($\bar{a}$, $\bar{b}$, $\bar{c}$) as edge is called unit vector.
- **Bravais Lattice :** Common nomenclature given to the 14 different lattices under the seven systems of the crystals.
- **Co-Ordination Number :** It is the number of nearest neighbours to a given atom in a crystal lattice.
- **Lattice Constant :** The distance between two neighbouring atoms, $a = \left(\frac{nM}{N\rho}\right)^{1/3}$.
- **Packing Factor :** The ratio of the volume of the atoms per unit cell to the total volume of the unit cell.
- **Lattice Plane :** A set of parallel equidistant planes passing through the lattice points.
- **Miller Indices :** $h : k : l = \frac{1}{p} : \frac{1}{q} : \frac{1}{r}$.

  The numbers **h**, **k**, *l* are called the Miller indices of a given set of planes and the plane is specified by (hk*l*).
- **Relation between Interplanar Distance and Miller Indices:**

  $$d_{(hkl)} = \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right)^{1/2}$$
- **Symmetry :** There exists certain operations which when operated upon a given structure, carry the structure into itself. The structure is said to possess symmetry under that operation.
- The main symmetry elements of a crystalline solid are :
  - Axes of symmetry.
  - Planes of symmetry.
  - Centre of symmetry.

- **Axis of Symmetry :** When a crystal is rotated about a vertical axis.
- **Plane of Symmetry :** When a crystal is cut along a certain plane and the mirror image of one half reproduces the other half of the crystal, then the plane along which it is cut, is called as plane of symmetry.
- **Centre of Symmetry :** It is a point in a crystal such that if a line is drawn from any point on the crystal through this point and produced an equal distance on the other side of this centre, it meets an identical point.
- **Bragg's Law :** $2d \sin \theta = n\lambda$.
- **Bragg's X-Ray Spectrometer :** Experimental arrangement for verification of Bragg's law.
- The X-rays are produced when high energy electrons are stopped by material having high atomic number.
- The X-ray beam contains continuous and characteristics X-rays.
- The frequency of an spectral line in characteristic X-ray spectrum varies directly as the square of the atomic number of the element emitting it.
- The time changing electric and magnetic fields are governed by Maxwell's equation.
- An electromagnetic wave travels with velocity C in vacuum.

IMPORTANT FORMULAE

- Lattice Constant, $a = \left(\frac{nM}{N\rho}\right)^{1/3}$
- Interplaner distance,

$$d_{(hkl)} = \frac{a}{\sqrt{b^2 + k^2 + l^2}}$$

- Bragg's Law, $2 d \sin \theta = n\lambda$
- For continuous X-rays,

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V} \text{ A}^\circ$$

- Moseley's law,

$$\upsilon \propto (Z - a)^2$$

- Maxwell's equations are

(i) $\nabla \cdot \vec{D} = \rho$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \cdot \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

(iv) $\nabla \cdot \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

UNSOLVED PROBLEMS

1. If density of NaCl is 2.163 g/cc and molecular weight is 58.45, find the spacing between planes parallel to the cubic lattice faces of NaCl crystal. [**Ans.** 2.83 A°]
2. A substance with fcc lattice has a density of 6250 kg/m^3 and molecular weight of 60.2. Calculate the lattice constant a. Given Avogadro number as 6.02×10^{26} per kg-mole. [**Ans.** 4 A°]
3. Find Miller indices of a set of parallel planes making intercepts in the ratio 3a : 4b on X, Y axes and being parallel to Z-axis, a, b, c, being primitive vectors of the lattice. [**Ans.** (4, 3, 0)]
4. Calculate interplanar spacing between (327) planes in a cubic crystal of lattice constant 4.2×10^{-8} cm.

   [**Ans.** 1.1×10^{-8} cm]
5. Calculate the wavelength of an X-ray beam incident at 12° for the first order reflection from calcite crystal if the grating constant of the crystal is 3.035 A°.

   [**Ans.** 1.26 A°]
6. Monochromatic X-rays of wavelength 1.5 A° are incident on a crystal whose interplanar spacing is 1.6 A°. Find the various orders in which Bragg's reflections take place.

   [**Ans.** 1st and 2nd order maxima]

EXERCISE

1. Define crystal lattice, basis and crystal structure.
2. What is meant by a space lattice ? How do we associate a crystal structure to the lattice ?
3. Describe the seven systems of crystals. Explain with examples the types of lattices in cubic systems.
4. What is co-ordination number ? Calculate co-ordination number for simple bcc and fcc lattices.
5. What is packing factor ? Show that the packing factor for simple cubic, bcc and fcc lattices are $\frac{\pi}{6}$, $\sqrt{3}\,\frac{\pi}{8}$ and $\sqrt{2}\,\frac{\pi}{6}$.
6. What is lattice constant ? Calculate for fcc and bcc lattices in terms of atomic radius.
7. What are Miller indices of a crystal ? Obtain an expression for the spacing between consecutive planes.
8. What are symmetry operations ? Enumerate the different types with examples.

9. Show the symmetry elements of an equilateral triangle and a square.
10. Explain and deduce Bragg's law in X-ray diffraction. Describe a Bragg's spectrometer and explain how it is used to determine the wavelength of X-rays.
11. What is X-ray diffraction ?
12. What are the types of X-rays? How they are produced?
13. Explain method of producing continuous spectra. Derive the formula for minimum wavelength of the X-rays.
14. State and explain Moseley's Law.
15. Write the Maxell's equation in different form.
16. Derive the formula for electromagnetic wave in free space.

UNIVERSITY QUESTIONS

December 2017

1. Define atomic radius. Calculate atomic radii in SC, BCC and FCC lattices with suitable diagrams. **[4+2]**

   Lead exhibits FCC structure. Each side of unit cell is of 4.95 A° Calculate radius of lead atom.

2. Derive the relation between interplaner spacing 'd defined by Miller Indices (hkl) and lattice parameter 'a'. **[4+2]**

   Calculate the interplaner spacing for (220) plane where the lattice constant is 4.938A°

3. What is X-ray? How do we get the continuous spectrum in X-rays explain. **[4+2]**
4. Derive an expression for electromagnetic wave in free space and hence calculate the value of velocity of light in free space. **[6]**

May 2018

1. Define Packing Density. Find the packing density in SC, BCC, and FCC lattices. **[6]**
2. Derive the relation between crystal density 'p' and lattice parameter 'a'.

   The density of copper is 8980 Kg/ m^3 and unit cell dimension is 3.61 A°. Atomic weight of copper is 63.54. Determine crystal structure. **[6]**

3. State and Derive Moseley's law for characteristics X-ray spectrum. **[6]**
4. What is displacement current? Write Maxwell's equations in differential and integral form. **[6]**

December 2018

1. What is primitive and nonpremitive unit cells? Find the number of atoms per unit cell in SC, BCC, FCC lattices. **[6]**
2. Define atomic radius. Find the atomic radius in SC, BCC, FCC lattices. **[6]**
3. State and Derive Bragg's law of X-ray diffraction. An X-ray is operated at 20 kV. Calculate the minimum wavelength of X-rays emitting from it. **[6]**
4. What is displacement current? Write Maxwell's equations in differential and integral form. **[6]**

May 2019

1. Define primitive and non-primitive unit cells. Calculate the lattice constant of iron which has BCC structure. Given ρ = 7.86 gm/cc, M = 55.85 **[6]**
2. Explain continuous X-ray spectrum with neat diagram, An X-ray is operated at 18 kV. Calculate the minimum velocity of electron bombarded at the anode. **[6]**
3. What is displacement current? Write Maxwell's equation in integral and differential form. **[6]**

* * *

UNIT V

MAGNETIC, SUPERCONDUCTING AND SEMICONDUCTING MATERIALS

5.1 INTRODUCTION TO MAGNETIC MATERIALS

- Magnetism finds applications in the understanding of electricity, optics, atomic structure etc. So, it has an important place in physics. Naturally found magnets are weak. However, for strong magnetic fields electromagnets are made use of. The earth itself is a huge and powerful magnet.
- The magnetic materials can be classified as paramagnetic, diamagnetic, ferromagnetic, ferrimagnetic and antiferro-magnetic.
- First we will define a few magnetic parameters and derive the relationship between them.

5.2 MAGNETIC PARAMETERS

- **Magnetic Field Strength 'H' :** The magnitude of the force experienced by an unit north pole at any point in the field is called the strength of the magnetic field at that point. The unit of field is gauss or oersted (C.G.S. unit). In M.K.S. system the intensity of magnetic field is expressed in Newton/Ampere-metre or weber/meter^2.
- **Magnetic Induction or Flux Density 'B' :** It is defined as the number of hypothetical induction lines passing normally through unit area. It is measured in Tesla (T) or weber/meter^2.

$\therefore$ $B = \phi/A$ where ϕ is the normal flux and A the area of cross-section.

- **Intensity of Magnetisation 'M' :** It measures the degree of magnetisation of a magnetised specimen and is defined as the magnetic moment per unit volume.

$$M = \frac{\text{Magnetic moment } (\mu)}{\text{Volume } (V)}$$

- **Magnetic Susceptibility 'χ' :** It measures the ease with which the specimen can be magnetised. It is defined as the ratio of the intensity of magnetisation induced in it to the magnetising field strength

$$\chi = \frac{M}{H}$$

- **Magnetic Permeability 'μ' :** The measure of the degree to which the lines of magnetic force can penetrate the medium is called the absolute permeability of the medium. It is denoted by μ_o.

  It is also defined as the ratio of the magnetic induction B produced in a material to the magnetising or induced field H.

$$\mu = \frac{B}{H}$$

Also $\mu = \mu_o \mu_r$

where μ_o is the permeability of free space and is equal to $4\pi \times 10^{-7}$ Henry/meter or is equal to 1 (in C.G.S. units), μ_r is the relative permeability which is measured by the ratio of the number of lines of force per unit area in the medium to the number of lines per unit area if the medium were replaced by vacuum. For free space $\mu_r = 1$ and $B = \mu_o H$.

- **Bohr Magneton** [May 18] **:** The magnetic moment of an electron is caused by its orbital or spin orbital momentum.
- The physical constant which represents this magnetic moment is called the **Bohr Magneton** and is represented by symbol μ_B.
- In SI system it is given by $\mu_B = \frac{e\hbar}{2m}$ and $\mu_B = \frac{e\hbar}{2mC}$ in CGS system.
- The magnitude of Bohr magneton is equal to 9.274×10^{-24} J/T.

Derivation :

The period of an electron orbiting in orbit of radius r is given by

$$T = \frac{2\pi r}{v} \qquad \ldots \text{I}$$

where v = velocity of electron

Due to the orbital motion, the current developed is given by,

$$I = \frac{-ev}{2\pi r} \qquad \ldots \text{II}$$

$$\therefore \quad \mu = \frac{-e\,v}{2\pi r}\cdot \pi r^2$$

$$\mu = \frac{-erv}{2}$$

Divide and multiply by m, mass of electron

$$\mu = \frac{-e}{2\,m}\cdot mvr$$

From Bohr's second postulate

$$mvr = \frac{n\,h}{2\pi}$$

$$\therefore \quad \mu = \frac{-e}{2m}\cdot\frac{n\,h}{2\pi}$$

$$\mu = -n\frac{eh}{4\pi m} \qquad \text{... IV}$$

The quantity $\frac{eh}{4\pi m}$ is called **Bohr Magneton** (μ_B)

∴ Bohr Magneton

$$\mu_B = \frac{e\,h}{4\pi M} \qquad \text{... V}$$

$$\text{or} \quad \mu_B = \frac{e\,\hbar}{2m} \qquad \text{... VI}$$

5.3 RELATIONSHIP BETWEEN μ AND χ

- Consider a magnetic material of cross-sectional area A and relative permeability μ_r to be placed in a uniform magnetic field of strength H.
- **Two Types of Lines of Induction Pass Through it :**

1. Due to the magnetic field.
2. Due to magnetisation by induction.

Hence, $B = \mu_o H + \mu_o M$ ... (A)

where μ_o is the permeability of free space.

But $B = \mu_o\,\mu_r\,H$ ... (B)

From (A) and (B)

$$\mu_o\,\mu_r\,H = \mu_o\,H + \mu_o\,M$$

$$\text{i.e.} \quad (\mu_r - 1)\,H = M$$

$\mu_r - 1 = \frac{M}{H} = \chi$ (χ being the susceptibility of the material).

5.4 TYPES OF MAGNETIC MATERIALS

- The most materials interact only slightly with an impressed magnetic field. A few substances, however, greatly alter any magnetic field in which they are placed.
- Consider a simple experiment to indicate the way in which material objects affect magnetic fields. An alternating potential difference is applied across the terminals of a toroidal coil so that current through it varies sinusoidally

$$i = i_o \sin 2\pi ft \qquad \text{... (5.1)}$$

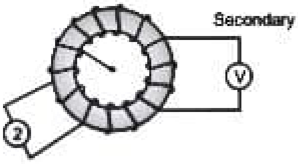


Fig. 5.1

- In Fig. 5.1 the voltage induced in the secondary coil can be used to determine the magnetic properties of the material inside the toroid.
- The flux inside such a coil is found to be

$$\phi = \frac{\mu_o\,N\,i\,a}{2}\,ln\left(1+\frac{a}{b}\right)$$

N is the total number of loops on the toroid. By Faraday's law, the induced e.m.f. in the secondary coil of N_2 loops wound on the toroid is

$$\epsilon_2 = -\frac{\mu_o\,N\,N_2\,a}{2\pi}\,ln\left(1+\frac{a}{b}\right)\frac{di}{dt}$$

$$\epsilon_2 = -M_\delta\frac{di}{dt} \qquad \text{... (5.2)}$$

where $M_\delta = \frac{\mu_o\,N\,N_2\,a}{2\pi}$ is the mutual inductance.

- Since we known how the current in the toroid changes with time, we can rewrite (5.2) as

$$\epsilon_2 = -2\pi\,f\,M_\delta\,i_o\cos 2\pi ft \qquad \text{... (5.3)}$$

- By using an appropriate voltmeter in the secondary coil, the maximum value of ϵ_2, namely, $2\pi f\,M_\delta\,i_o$ can be measured.
- In obtaining (5.3) it has been assumed that the space in the interior of the coil is empty.

$$\therefore \quad \mu = \frac{-e\,v}{2\pi r}\cdot \pi r^2$$

$$\mu = \frac{-erv}{2}$$

Divide and multiply by m, mass of electron

$$\mu = \frac{-e}{2\,m}\cdot mvr$$

From Bohr's second postulate

$$mvr = \frac{n\,h}{2\pi}$$

$$\therefore \quad \mu = \frac{-e}{2m}\cdot\frac{n\,h}{2\pi}$$

$$\mu = -n\frac{eh}{4\pi m} \qquad \text{... IV}$$

The quantity $\frac{eh}{4\pi m}$ is called **Bohr Magneton** (μ_B)

∴ Bohr Magneton

$$\mu_B = \frac{e\,h}{4\pi M} \qquad \text{... V}$$

$$\text{or} \quad \mu_B = \frac{e\,\hbar}{2m} \qquad \text{... VI}$$

5.3 RELATIONSHIP BETWEEN μ AND χ

- Consider a magnetic material of cross-sectional area A and relative permeability μ_r to be placed in a uniform magnetic field of strength H.
- **Two Types of Lines of Induction Pass Through it :**

1. Due to the magnetic field.
2. Due to magnetisation by induction.

Hence, $B = \mu_o H + \mu_o M$ ... (A)

where μ_o is the permeability of free space.

But $B = \mu_o \mu_r H$ ... (B)

From (A) and (B)

$$\mu_o \mu_r H = \mu_o H + \mu_o M$$

i.e. $(\mu_r - 1)H = M$

$\mu_r - 1 = \frac{M}{H} = \chi$ (χ being the susceptibility of the material).

5.4 TYPES OF MAGNETIC MATERIALS

- The most materials interact only slightly with an impressed magnetic field. A few substances, however, greatly alter any magnetic field in which they are placed.
- Consider a simple experiment to indicate the way in which material objects affect magnetic fields. An alternating potential difference is applied across the terminals of a toroidal coil so that current through it varies sinusoidally

$$i = i_o \sin 2\pi ft \qquad \text{... (5.1)}$$

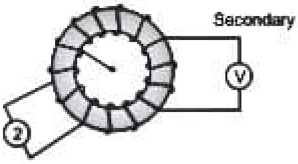


Fig. 5.1

- In Fig. 5.1 the voltage induced in the secondary coil can be used to determine the magnetic properties of the material inside the toroid.
- The flux inside such a coil is found to be

$$\phi = \frac{\mu_o N\,i\,a}{2}\,ln\left(1+\frac{a}{b}\right)$$

N is the total number of loops on the toroid. By Faraday's law, the induced e.m.f. in the secondary coil of N_2 loops wound on the toroid is

$$\epsilon_2 = -\frac{\mu_o N N_2 a}{2\pi}\,ln\left(1+\frac{a}{b}\right)\frac{di}{dt}$$

$$\epsilon_2 = -M_\delta\frac{di}{dt} \qquad \text{... (5.2)}$$

where $M_\delta = \frac{\mu_o N N_2 a}{2\pi}$ is the mutual inductance.

- Since we known how the current in the toroid changes with time, we can rewrite (5.2) as

$$\epsilon_2 = -2\pi f M_\delta i_o \cos 2\pi ft \qquad \text{... (5.3)}$$

- By using an appropriate voltmeter in the secondary coil, the maximum value of ϵ_2, namely, $2\pi f M_\delta i_o$ can be measured.
- In obtaining (5.3) it has been assumed that the space in the interior of the coil is empty.

- Experiments can be performed when the interior of the coil is completely filled with some material, and the effect on-the induced voltage $\in_2$ caused by changing the material within the coil can be found. It is found that the value of $\in_2$ does not change if the coil is filled with such very diverse substances as air, water, oil, wood, Al, Cu, plastic etc.
- Only a very special class of substances, change the induced voltage, by large amount. These are the so called ferromagnetic materials, and they are almost always pure or alloy composition containing iron, cobalt and nickel.
- In 1845 Faraday concluded that all substances were, to a greater or lesser extent affected by a magnet a few being attracted, but most are repelled. This gave rise to the grouping of different substances into
  - ➢ Ferromagnetic like iron, which are strongly attracted
  - ➢ Paramagnetic, feebly attracted and
  - ➢ Diamagnetic which are repelled by a magnet.

  Materials can also be classified on the following basis :

  (i) Sign of χ and its value.

  (ii) Value of permeability μ_a.

  (iii) Presence or absence of permanent magnetic dipoles.

5.4.1 Classification on the Basis of χ and μ_a

- **Paramagnetic :** Substances are those for which I varies linearly with H and χ has a small positive value. Also μ_a is slightly greater than 1. Examples are, platinum solutions of salts of iron, oxygen, manganese, palladium etc. For platinum $\mu = 1.00002$ and $\chi = 1.71 \times 10^{-6}$. Also, χ not only decreases with increase of magnetising force but it also depends on the temperature. Curie discovered that the susceptibility of some paramagnetics varies inversely as the absolute temperature.
- **Diamagnetic :** Substances have $\mu < 1$ and χ is constant and has a negative value of the order of 10^{-4} to 10^{-6}. Example, bismuth, antimony, Zn, Ag, Cu, Sb, Au, Fb, water, alcohol, air hydrogen. When placed in a magnetic field they have a tendency to move away from the field.
- **Ferromagnetic :** Substances are those which can be magnetised to a great extent. They have an abnormally high value of χ e.g. steel, iron, cobalt, nickel and alloys of these substances. In these substances the magnetisation is not proportional to the magnetising force, hence χ and μ vary with the magnetising force considerably. Also χ varies with temperature. When a ferromagnetic substance is heated, its χ varies inversely as the absolute temperature. This is called as **Curies Law** and is expressed as $\chi\ T = \text{Constant}$ (T being absolute temperature). Thus, χ steadily decreases with increase in temperature, until a critical temperature is reached, at which ferromagnetism disappears and the substance becomes paramagnetic. This absolute temperature is called the **Curie Temperature**. The susceptibility of a ferromagnetic substance above its Curie point is inversely proportional to the amount its temperature is above the Curie temperature.

  i.e. $\chi \propto \dfrac{1}{(T - T_c)}$, T_c being Curie temperature.

  This law is called the **Curie - Weiss Law**.

  T_c for cobalt is about 1100°C, for nickel 4000°C and for iron 770°C.

  Ferromagnetics have non linear variation of μ_a with H i.e. $B \neq \mu_o H$ and hysteresis effect is exhibited.

5.4.2 Classification on the Basic of Presence or Absence of Permanent Magnetic Dipoles

[May 18]

- The magnetic materials can also be classified on-the basis of the presence or absence of permanent magnetic dipoles in them. The materials which lack permanent magnetic dipoles are called **Diamagnetic**. The magnetisation of such materials occurs when the applied field induces a magnetic moment in the individual atoms. Due to the external magnetic field causing changes in the electron orbits. In the absence of the external field, the net magnetic moment of the orbit is zero.
- If permanent dipoles are present in the atoms of a material, it could be **Paramagnetic**, **Ferromagnetic**, **Antiferromagnetic** or **Ferrimagnetic**. The differentiation is done on the basis of interaction between individual dipoles.
  - ➢ The substance is diamagnetic if dipole interaction is zero and orientation of individual dipole moment is random. Point the magnetic moments will align with an applied magnetic field.
  - ➢ For ferromagnetics, dipoles interact to line up parallel to each other, to give a large resultant magnetisation. This is known as ordered magnetism because of the stronger interatomic interactions.

- When magnetic moments of equal magnitude are present and neighbouring moments are aligned antiparallel, the substance is antiferromagnetic. In these substances, the magnetisation vanishes.
- When unequal magnetic moments are present-and are aligned antiparallel to each otter, the substance is ferrimagnetic. In these substances a net magnetisation exists.

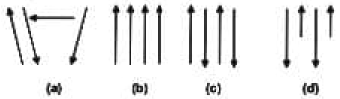


Fig. 5.2 : Paramagentic, ferromagnetic, antiferromagnetic and ferrimagnetic arrangement of spins

- Diamagnetism is a universal property of all materials. However, diamagnetic properties are weaker than paramagnetic ones and still weaker than ferromagnetic properties. The presence of a permanent magnetic moment in atoms is a necessary condition for the existence of ferromagnetic properties.
- The pecularities of ferromagnetism are due to the formation of vast regions or domains. In the domains the magnetic moments of a large number of atoms are arranged parallel to one another giving magnetic saturation.

5.4.3 Ferrites

[Dec. 18, May 19]

- The ferrites are a type of ceramic compound composed of (Fe_2O_3) combined chemically with one or more additional metallic elements. They are basically ferrimagnetic i.e. they can be magnetised or attracted by a magnet and are electrical insulator.
- The ferrites have a spiral crystal structure and the chemical formula for the ferrites are given as $X\ Y_2\ Z_4$ in which X is a divalent negative ion. Y is Fe and Z is mostly the divalent oxygen atom.
- The most common ferrite is Fe_3O_4 whose chemical formula can be written as $Fe^{2+}\ Fe_2^{3+}\ O_4^{2-}$. The other divalent metallic ions used are Co^{2+}, Mn^{2+}, Zn^{2+}, Cd^{2+} etc.
- The ferrites have wide applications in electrical engineering. The most common application of ferrites are
  - Hard ferrites are used in permanent magnets.
  - They are also used in transformer core.
  - As ferromagnetic insulators in electrical circuits.
  - In type recorder head for recording.
  - The main disadvantage is that they have low electrical resistivity.

5.4.4 Garnets

[Dec. 18, May 19]

- Garnets are the group of silicate materials that are being used as gemstone and abrasive. The different types of garnets have similar properties and crystal forms but have different chemical composition.
- The garnets are found in colors like red, orange, yellow, green, etc of which the red color is most common.
- The garnets are nesosilicate having the formula $X_3Y_2\ (SiO_4)_3$ where the X site is occupied by divalent cations Ca^{2+}, Mg^{2+}, Mn^{2+} and Y by trivalent cations like Al^{3+}, Fe^{3+}, Cr^{3+}.
- The garnets are crystalline in cubic system having three axis that are of equal length and perpendicular to each other.
- The garnets do not show cleavage so under pressure when they fracture irregular pieces are formed.

5.5 HYSTERESIS LOOP (B – H CURVE)

[Dec. 17]

- Consider an unmagnetised bar of a ferromagnetic substance. Subject it to a magnetising field as follows :
- To start with, gradually increase the magnetising field H and find the corresponding value of I (the intensity of magnetisation) or of B (the magnetic induction).
- A graph of B verses H gives a curve OAC as shown in Fig. 5.3. If we increase the magnetising force beyond the point C, B remains constant and the substance is said to be **Saturated**.

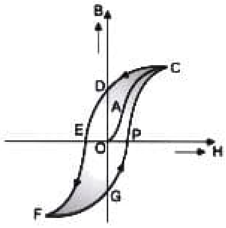


Fig. 5.3 : Cycle of magnetisation H-I curve for a ferromagnetic material

- After reaching the point C, gradually decrease, the H and obtain the value of B (as H decreases). It is found that the curve obtained does not coincide with that obtained with increasing value of H. It takes the form CD.

- Thus when the magnetising force is zero, the intensity of magnetisation (or the induction B) instead of being zero has a value = OD.
- This value of intensity of magnetisation for which H = 0 is called **Residual Magnetism** or **Retentivity** or **Remanence**.
- If the direction of H is reversed, the curve DEF is obtained. On decreasing H to zero and then increasing in its original direction the curve FGPC is obtained.
- Thus in all cases I (the intensity of magnetisation) or B (the induction) appears to lag behind the magnetising force.
- This lagging of B (or I) behind the magnetising force is called **Hysteresis**. The loop so obtained is called the **Hysteresis Loop**. The cycle of operation is called **Hysteresis Cycle**. The magnetising force represented by OE or OG represents the force required to remove the residual magnetism of the bar. Therefore it gives the **Coercive Force** for the material.
- The shape of the hysteresis loop between B and H is similar to the one between I and H. The areas of loops are different in both the cases.
- In B - H curve the intercept on the B-axis is called **Remanent** induction. In this curve the value of the applied field to make B = 0 is not coercivity because

$$B = H + 4\pi I \text{ (Using the relation in CGS system)}$$

When $B = 0$

$$H = -4\pi I$$

As I is not zero the specimen is magnetised.

- The hysteresis loop obtained depends entirely on the absence of mechanical vibrations. The mechanical vibrations tend to destroy the retentivity and this results in the partial or complete coincidence of the two sides of the hysteresis loop. The shape of hysteresis loop is a characteristic of the magnetic material.
- Fig. 5.4 shows the shapes B and A in the case of steel and soft iron respectively. From the curves it is seen that soft iron has greater retentivity than steel but less coercivity or in other words, steel retains magnetism
ft iron loses it.

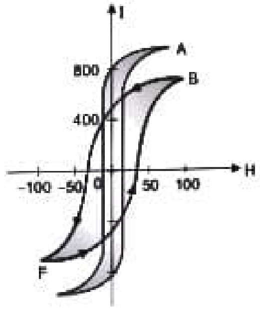


Fig. 5.4 : Hysteresis curve : A – soft Iron, B – Hard steel

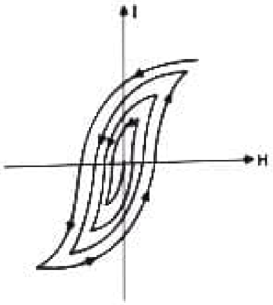


Fig. 5.5 : Demagnetisation

Demagnetisation :

- In order to demagnetise a substance, it must be taken through a cycle of magnetisation with gradually decreasing magnetising force (Fig. 5.5). To achieve this, insert the bar in a coil through which an alternating current of gradually diminishing value is passed.
- Demagnetisation can also be achieved if the substance is heated above the critical temperature.

5.6 INTRODUCTION TO CONDUCTING MATERIALS

- Solids differ from each other in their physical properties like electrical, optical, mechanical etc. It is desired to understand these physical properties for scientific and engineering application of the material. One of the most important properties from engineering point of view is electrical property. So in this part, we will mainly concentrate on electrical properties.
- With respect to electrical properties, metals are good **Conductors** whereas other solids can be classified as either **Semiconductors** or **Insulators**.
  - ➢ In conductors, the electrons are free to move within the specimen. These free electrons are contributed by the atoms within the specimen.

- As the electron moves away from the atom, the atom becomes positive ion. These **Positive Ions** or **Lattice Points** are fixed at a particular point and vibrate back and forth from its mean position. Higher the temperature, more will be the amplitude of vibration.
- Under the influence of electric field, the electrons move freely. But their velocity is reduced with collisions with lattice points. The conducting properties of material is governed by these free electrons and conductivity is reciprocal of resistivity.
- At room temperature, the conductivity of conductors ranges from 10^6 to 10^8 mho/m. The conductivity range of semiconductors and insulators is from 10^{-6} to 10^4 mho/m and from 10^{-16} mho/m to 10^{-7} mho/m respectively.

5.6.1 Free Electron Theory of Metals

- The free electron theory of metals was first proposed by Drude and later improved by Lorentz and hence the theory is called the Drude-Lorentz theory. Following are the basic assumptions made in the theory :
  - All metals contain a fixed number of valence electrons forming an **Electron Gas**, which are free to move throughout the volume of the metal.
  - The electron velocities in metals obey the classical **Maxwell-Boltzmann Distribution** of velocities.
  - The positive ions which can vibrate about their mean position, cannot move from one lattice site to another. The repulsive force between the negatively **Charged Electron** is ignored and the electric field due to the positive ions is assumed to be uniform.
  - The electrons move from one point to another randomly with **Random Velocity** which is temperature dependent. At room temperature, this velocity is about 4×10^5 m/s.
  - The **Kinetic Energy** of the electron is given by 3 kT/2, where k is Boltzmann's constant and T is absolute temperature.
  - In absence of external electric field, the electrons move in **Random Directions**, making collisions from time to time with positive ions, which are fixed in lattice. This makes net current zero.
  - When an electric field is applied, free electrons move towards positive terminal of the supply. Thus, the electrons will experience two motions – random motion due to temperature and drift motion due to applied voltage. As a result the electron will move in **Opposite Direction to the Electric Field** while maintaining their random motion.
  - While drifting towards positive of the supply, the electrons colloid with positive ions. During each collision the electron loses all its drift velocity and starts from rest once again. The average distance covered by an electron between collisions is known as **Mean Free Path** 'λ' and time taken to cover this distance is termed as relaxation time 'τ'.
  - As the temperature increases, the vibration of the ion core increases, this increases the probability of electron-core collision. As a result, **Resistivity Increases with Increase in Temperature.**
- By replacing the classical statistics by Fermi-Dirac statistics, Sommerfeld calculated the conductivity along the line of Lorentz's theory. At equilibrium the free electrons have different velocities. In the absence of electric field, the velocities are in all directions and the velocity vectors cancel each other and net velocity vector is zero.
- The velocity of the electron present in the Fermi level is called Fermi velocity. When the electric field is applied along X-axis, the electron starts **Drifting with Velocity** v_x and the force experienced by the electron is eE. The forces on the electrons are governed by the equation,

$$ma = eE \qquad \text{... (5.4)}$$

$$\therefore \quad m\frac{dv_x}{dt} = eE$$

$$\therefore \quad dv_x = \frac{eE}{m}dt$$

$$\therefore \quad v_x = \frac{dx}{dt} = \frac{eE}{m}t + k \qquad \text{... (5.5)}$$

where k is constant of integration.

At $t = 0, v_x = 0.$

$$\therefore \quad k = 0$$

Substituting in equation (5.5),

$$v_x = \frac{eE}{m}t \qquad \text{... (5.6)}$$

The average drift velocity is given by,

$$\bar{v}_x = \frac{eE}{m}\tau \qquad \text{... (5.7)}$$

where, τ = relaxation time

$$\tau = \frac{\lambda}{v_x}$$

where, λ = mean free path

Thus, the average drift velocity is proportional to the applied electric field as $\frac{e\tau}{m}$ is constant. The constant $\frac{e\tau}{m}$ is called **Drift Mobility** μ.

$$\therefore \quad \bar{v}_x = \mu E$$

$$\therefore \quad \mu = \frac{\bar{v}_x}{E} \qquad \text{... (5.8)}$$

Therefore, the drift velocity is defined as the increase in the average electron velocity per unit of electric field.

The electrical current density is given by

$$J = ne\,\bar{v}_x \qquad \text{... (5.9)}$$

where n is the number of electrons per unit volume.

From equation (5.7),

$$J = ne\frac{eE}{m}\tau$$

$$J = \left(\frac{n\,e^2\,\tau}{m}\right)E \qquad \text{... (5.10)}$$

The Ohm's law is given by

$$J = \sigma E \qquad \text{... (5.11)}$$

Comparing (5.10) and (5.11), we get electrical conductivity.

$$\sigma = n\frac{e^2\,\tau}{m} \qquad \text{... (5.12)}$$

The mobility is given by

$$\mu = \frac{\bar{v}_x}{E} = \frac{e\,E\,\tau}{mE} = \frac{e\tau}{m} \text{ using equation (5.7)} \qquad \text{... (5.13)}$$

$$\therefore \quad \sigma = ne\mu \qquad \text{... (5.14)}$$

The expression is same as that obtained on the basis of classical theory. As charge is constant, the conductivity depends on charges per unit volume n and their mobility μ.

- Both the classical and the quantum theories led to the same expressions, there is an essential difference in their approaches. According to the classical theory all free electrons contribute to the electrical conduction whereas according to the quantum theory only those electrons near the Fermi level take part in the electrical conduction.
- The quantum free electron theory is successful in explaining many properties of metals like specific heat, electrical and thermal conductivities, magnetic susceptibility etc. The main drawback of this theory of that it has failed to explain why some solids are semiconductors while some others are insulators and why divalent metals have lower conductivities than monovalent metals and why some metals exhibit positive Hall coefficient.

5.6.2 Drawbacks of Classical Free Electron Theory

- The free electron theory, successfully established Ohm's law, showed that the resistivity is directly proportional to temperature and the Wiedemann-Franz relation was proved. However, the theory has many drawbacks.

The main drawbacks are :

- ➢ The specific heat capacity value based on classical theory shows that it is independent of temperature. But as per quantum theory, it directly depends on temperature i.e. it increases with the increase in temperature.
- ➢ As per classical theory, the paramagnetic susceptibility is inversely proportional to temperature. But experimental results show that it is almost independent of temperature.
- ➢ The classical theory failed to explain occurrence of long mean free paths (10^8 or 10^9 times interatomic spacing).
- ➢ Classification of solids i.e. metals, semimetals, semiconductors and insulators cannot be done by classical theory.
- ➢ The positive values of Hall coefficient of metals could not be explained by classical theory.
- ➢ Classical theory also failed to explain photoelectric effect, Compton effect and black body radiation.

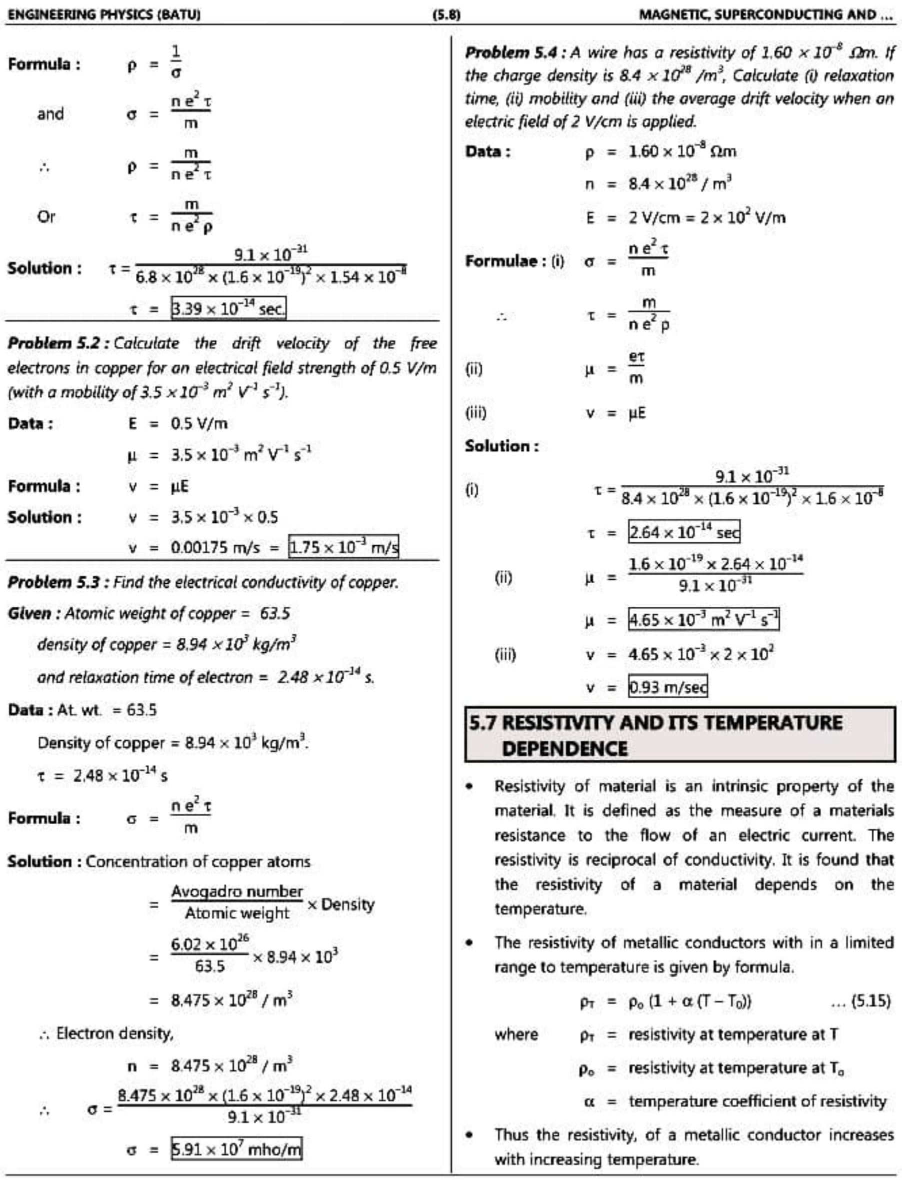
SOLVED PROBLEMS

Problem 5.1 : *Find the relaxation time of conduction electrons in a metal having resistivity 1.54×10^{-8} Ωm and electron density 6.8×10^{28} m^3.*

Data :

$$\rho = 1.54 \times 10^{-8}\ \Omega\text{m}$$

$$n = 6.8 \times 10^{28} / \text{m}^3$$



Formula : $\rho = \frac{1}{\sigma}$

and $\sigma = \frac{n e^2 \tau}{m}$

$\therefore \quad \rho = \frac{m}{n e^2 \tau}$

Or $\tau = \frac{m}{n e^2 \rho}$

Solution : $\tau = \frac{9.1 \times 10^{-31}}{6.8 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.54 \times 10^{-8}}$

$\tau = \boxed{3.39 \times 10^{-14} \text{ sec.}}$

Problem 5.2 : *Calculate the drift velocity of the free electrons in copper for an electrical field strength of 0.5 V/m (with a mobility of* $3.5 \times 10^{-3}\ m^2\ V^{-1}\ s^{-1}$*).*

Data : $E = 0.5 \text{ V/m}$

$\mu = 3.5 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$

Formula : $v = \mu E$

Solution : $v = 3.5 \times 10^{-3} \times 0.5$

$v = 0.00175 \text{ m/s} = \boxed{1.75 \times 10^{-3} \text{ m/s}}$

Problem 5.3 : *Find the electrical conductivity of copper.*

Given : *Atomic weight of copper = 63.5*

density of copper = $8.94 \times 10^3\ kg/m^3$

and relaxation time of electron = 2.48×10^{-14} *s.*

Data : At. wt. = 63.5

Density of copper = $8.94 \times 10^3 \text{ kg/m}^3$.

$\tau = 2.48 \times 10^{-14} \text{ s}$

Formula : $\sigma = \frac{n e^2 \tau}{m}$

Solution : Concentration of copper atoms

$$= \frac{\text{Avogadro number}}{\text{Atomic weight}} \times \text{Density}$$

$$= \frac{6.02 \times 10^{26}}{63.5} \times 8.94 \times 10^3$$

$$= 8.475 \times 10^{28} / \text{m}^3$$

$\therefore$ Electron density,

$n = 8.475 \times 10^{28} / \text{m}^3$

$\therefore \quad \sigma = \frac{8.475 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 2.48 \times 10^{-14}}{9.1 \times 10^{-31}}$

$\sigma = \boxed{5.91 \times 10^7 \text{ mho/m}}$

Problem 5.4 : *A wire has a resistivity of* $1.60 \times 10^{-8}\ \Omega m$*. If the charge density is* $8.4 \times 10^{28}\ /m^3$*, Calculate (i) relaxation time, (ii) mobility and (iii) the average drift velocity when an electric field of 2 V/cm is applied.*

Data : $\rho = 1.60 \times 10^{-8}\ \Omega\text{m}$

$n = 8.4 \times 10^{28} / \text{m}^3$

$E = 2 \text{ V/cm} = 2 \times 10^2 \text{ V/m}$

Formulae : (i) $\sigma = \frac{n e^2 \tau}{m}$

$\therefore \quad \tau = \frac{m}{n e^2 \rho}$

(ii) $\mu = \frac{e\tau}{m}$

(iii) $v = \mu E$

Solution :

(i) $\tau = \frac{9.1 \times 10^{-31}}{8.4 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 1.6 \times 10^{-8}}$

$\tau = \boxed{2.64 \times 10^{-14} \text{ sec}}$

(ii) $\mu = \frac{1.6 \times 10^{-19} \times 2.64 \times 10^{-14}}{9.1 \times 10^{-31}}$

$\mu = \boxed{4.65 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}}$

(iii) $v = 4.65 \times 10^{-3} \times 2 \times 10^2$

$v = \boxed{0.93 \text{ m/sec}}$

5.7 RESISTIVITY AND ITS TEMPERATURE DEPENDENCE

- Resistivity of material is an intrinsic property of the material. It is defined as the measure of a materials resistance to the flow of an electric current. The resistivity is reciprocal of conductivity. It is found that the resistivity of a material depends on the temperature.
- The resistivity of metallic conductors with in a limited range to temperature is given by formula.

$$\rho_T = \rho_0 (1 + \alpha (T - T_0)) \quad \ldots (5.15)$$

where ρ_T = resistivity at temperature at T

ρ_0 = resistivity at temperature at T_o

α = temperature coefficient of resistivity

- Thus the resistivity, of a metallic conductor increases with increasing temperature.

In terms of charge density n, the resistivity is given by,

$$\rho = \frac{m}{ne^2\tau} \quad \text{... (5.16)}$$

where, n = charge density

τ = the average time between collision.

- In metals the charge density 'n' does not changes with temperature. However the increase in temperature can increase the collision of electrons. This reduce τ and implies that increase in temperature increase the resistivity.
- However in insulators and semiconductors the charge density 'n' increases with the increasing temperature. Thus an increase in temperature decreases the resistivity.
- The resistivity of semiconductor is given by,

$$\rho = \frac{1}{n_i e (\mu_e + \mu_n)} \quad \text{... (5.17)}$$

where n_i = charge density in intrinsic semiconductors.

- In intrinsic semiconductors the carrier concentration n_i increases with temperature as

$$n_i^2 = A_o T^3 e^{-Eg/KT} \quad \text{... (5.18)}$$

where, A_o = Constant independent of temperature

E_g = Energy gap

T = Absolute temperature

Thus the resistivity of the semiconductor decreases with increase in the temperature.

5.8 MICROSCOPIC OHM'S LAW [May 18]

- When electric current in a material is proportional to the voltage across it, the material is said to be **Ohmic**, or to obey **Ohm's Law**. A microscopic view suggest that this proportionality comes from the fact that an applied electric field superimposes a small drift velocity on the free electrons in a metal.
- For ordinary currents, this drift velocity is of the order of millimeters per second in contrast to the speeds of electrons them selves which are of the order of 10^6 m/s. Even by the electron speeds are small, the speed of transmission of electric signal along a wire is very high.
- The current density can be expressed as

$$I = ne\,v\,A$$

or
$$J = \frac{I}{A} = ne\,v \quad \text{... (5.19)}$$

But,
$$I = \frac{V}{R}$$

$\therefore$
$$J = \frac{V}{RA}$$

as
$$R = \frac{\rho L}{A}$$

$\therefore$
$$J = \frac{V}{\frac{\rho L}{A} \cdot A}$$

$$J = \frac{EL}{\rho L} \quad \text{...}\left(\because E = \frac{V}{L}\right)$$

$$J = \frac{F}{\rho}$$

$\therefore$
$$J = \sigma E \quad \text{... (5.20)}$$

This is microscopic Ohm's Law.

5.9 INTRODUCTION TO SUPERCONDUCTIVITY [Dec. 17]

- It is a known fact that the resistivity of pure metals decreases with decreasing temperature. When the temperature falls below a certain value (the exact value depending on the substance), the resistivity vanishes entirely.

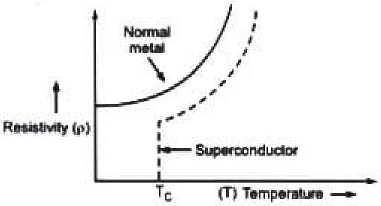


Fig. 5.6 : Variation of resistance with temperature

- In metals, both the thermal vibrations of atoms and the presence of impurities or imperfections scatter the moving conduction electrons. This gives rise to electrical resistivity. The variation of resistivity for a pure metal and superconductor is shown in Fig. 5.6.
- At the beginning of the twentieth century, in 1908, H. Kamerling Onnes, a Dutch Physicist, successfully liquified helium. As helium boils at 8.2 K, it therefore became possible to study the properties of materials at low temperature.

- In 1911, he observed that the electrical resistivity of pure mercury dropped suddenly to zero at about the boiling point of helium. He concluded that mercury had passed into a new state, which he called the **Superconducting State** due to its remarkable electrical properties.
- The temperature at which the material changes its state from a state of normal resistivity to a superconducting state, is called the **Transition or Critical Temperature T_c.**
- A conductor having zero (or almost zero) electrical resistance is called a **Superconductor** and this phenomenon is called as **Superconductivity**.

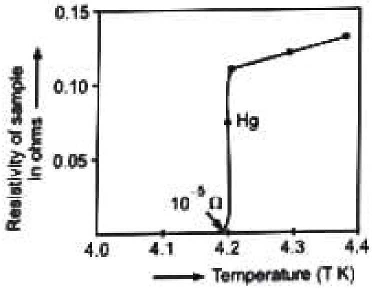


Fig. 5.7: Resistance of mercury as a function of temperature showing a transition from normal state to superconducting state at a critical temperature of 4.2 K

- The superconducting transition is found to be very sharp for a pure metal and it is broad for an impure metal. The zero magnetic induction in a superconductor is responsible for levitation effects.
- In a famous levitation experiment, a horizontal bar magnet was suspended from a chain. It was lowered over a sheet of lead, which had been cooled to the superconducting state. As the magnet came nearer to the superconducting state, the magnet remained floating horizontally over the lead sheet.
- The field of the approaching magnet induces a current on the surface of the superconductor. As the resistance is zero in the superconductor, the current persisted and the field due to the current repelled the bar magnet.
- This persistence of currents is found uniquely in superconductors. Certain experiments on the study of decay of these supercurrents in a solenoid found decay time to be greater than 10^5 years.

5.10 PROPERTIES OF SUPERCONDUCTORS

Following are the properties of superconductors:

5.10.1 Zero Electrical Resistance

- A superconductor is characterized by zero electrical resistance. The temperature below which the resistance of the material vanishes is called as the **Transition Temperature** or **Critical Temperature**. It is referred as T_c.
- As it is not possible to test experimentally whether the resistance is zero, the specimen is connected in a circuit as shown in Fig. 5.8.

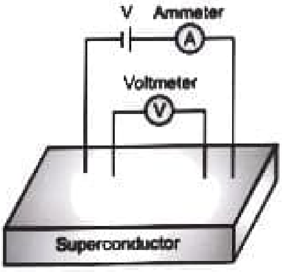


Fig. 5.8

- When the material is in normal conducting state, a voltage drop is measured across its ends. As the material is cooled below its transition temperature T_c, the voltage drop disappears as its resistance drops to zero (R = V/I).
- A more sensitive method devised by K. Onnes consists in measuring the decrease of current in a closed ring of superconducting wire.

Table 5.1 : A List of Some Superconductors along with their Critical Temperature

Sr. No.	Material	T_c in K
1.	Copper, silver, gold	Non-superconducting
2.	Rhodium	240×10^{-6}
3.	Aluminium	1.1
4.	Tin	3.72
5.	Mercury	4.15
6.	Lead	7.2
7.	Niobium	9.3
8.	Niobium-titanium alloys	9-11
9.	Lead molybdenum sulphide	14
10.	Niobium-tin	18.3
11.	Vanadium-gallium	15.4
12.	Niobium-germanium	23.3

- It has been observed that traces of paramagnetic elements in the specimen can lower the transition temperature. Hence, it becomes necessary to remove these traces completely. Non-magnetic impurities have no marked effect on the transition temperature.

5.10.2 Critical Field: Effect of External Magnetic Field

- K. Onnes discovered in 1913 that, when a superconductor is placed in an increasing magnetic field, it loses superconductivity at a certain value H_c of the field. The magnetic field strength at which superconductivity gets destroyed is called the **Critical Magnetic Field** H_c. This value is a characteristic of the metal and depends on its orientation in the magnetic field and the temperature.
- The relation between superconductivity and magnetic field plays an important role in the study of properties of superconductors. Obviously, the value of H_c varies with temperature. Fig. 5.9 shows the variation of H_c with temperature for a typical superconductor.

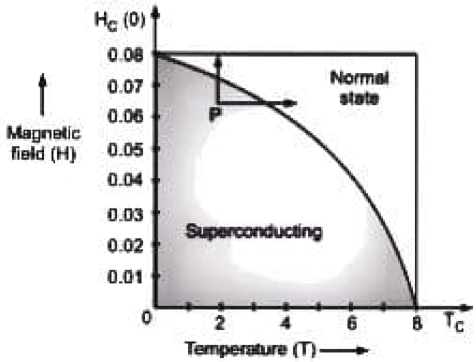


Fig. 5.9

- From Fig. 5.9, consider point P, where the temperature and the magnetic field are within the shaded region, the metal is in the superconducting state. On increasing either the temperature or the field, it can be driven into the normal state. Hence, it can be seen that a superconductor has two possible states: (i) The superconducting one which is resistanceless and perfectly diamagnetic and (ii) A normal state which is the same as a normal metal.
- At any temperature $T < T_c$, the material remains superconducting until a corresponding critical magnetic field is applied. When the magnetic field exceeds the critical value, the material goes into the normal state. The critical field required to destroy the superconducting state decreases progressively with increase in temperature.
- For example, a magnetic field of 0.04 T will destroy the superconductivity of mercury at $T \approx 0$ K, whereas a field of 0.02 T is sufficient to destroy its superconductivity at $T \approx 3$ K.
- The variation of critical field with temperature is given by the relation

$$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)\right]^2$$

where $H_c(0)$ is the critical magnetic field at 0 K.

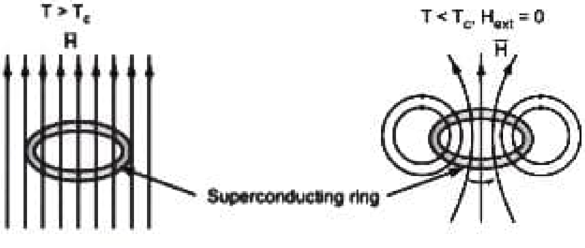
5.10.3 Persistent Currents

- Consider a superconducting ring placed in a magnetic field. When cooled to below the critical temperature, it becomes superconducting. The external field induces a current in the ring. When switched off, the current will continue to keep flowing, on its own accord, around the loop, as long as the loop is held below the critical temperature.
- Such a steady current flowing with undiminished strength is called **Persistent Current**. This current does not need external power to maintain it as there does not exist I^2R losses. If the superconducting ring has a finite resistance R, the current circulating in the ring would decrease according to the relation,

$$I(t) = I(0)\ e^{-Rt/L}$$

where L is the inductance of the ring.

- Calculations show that once the current flow is initiated, it persists for more than 10^5 years. Persistent current is one of the most important properties of a superconductor.
- Superconductor coils with persistent currents produce magnetic fields. They can therefore be used as magnets which do not require a power supply to maintain its magnetic field.



(a) Superconducting ring is cooled in the presence of a magnetic field

(b) At $T < T_c$, the magnetic field is switched off. Persistent current is induced in the ring

Fig. 5.10

5.10.4 Critical Current Density (J_c)

- The magnetic field which destroys superconductivity, need not be due to an externally applied field, but it may be the field produced as a result of current flow in the superconductor ring itself. If the field produced by itself exceeds H_c, the superconductivity of the ring is destroyed.
- Thus, if a superconducting material carries a current and if the magnetic field produced by it is equal to H_c, then superconductivity disappears. The maximum current density J at which superconductivity vanishes is called the **Critical Current Density** J_c. For $J < J_c$, the current can sustain itself while for $J > J_c$, the current cannot sustain itself. A superconducting ring of radius R loses its superconductivity when the current is,

$$I_c = 2\pi R H_c$$

∴ The critical current density,

$$J_c = \frac{\text{Critical current}}{\text{Area of the ring}}$$

$$J_c = \frac{2\pi R H_c}{\pi R^2} = \frac{2H_c}{R}$$

This sets a limit to the maximum current a superconductor can carry without disturbing its superconducting state.

- As the temperature is raised, the maximum current that a superconductor can carry decreases as the temperature is raised and falls to zero at the transition temperature T_c. This maximum current leads to a maximum applied magnetic field.

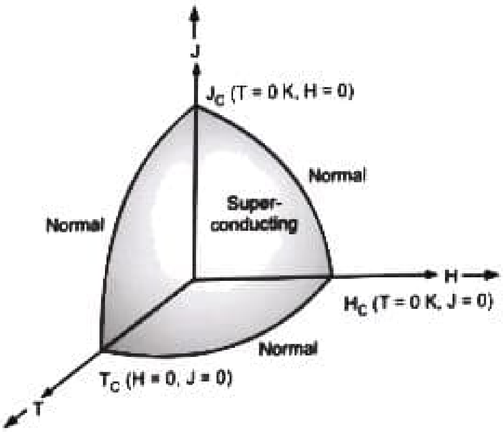


Fig. 5.11

- As critical current falls with the temperature, the critical magnetic field will also decrease as the transition temperature is approached. The variation of critical current density J_c and critical magnetic field H_c with temperature is shown in Fig. 5.11.
- Fig. 5.11 shows the combined effects of temperature, current density and magnetic field on a superconductor. The boundary separates superconducting and normal states. Within the boundary, the state is superconducting.

  In the superconducting state,

$$T < T_c$$
$$H < H_c$$
$$\text{and } J < J_c$$

5.11 MEISSNER EFFECT [Dec. 17, 18, May 19]

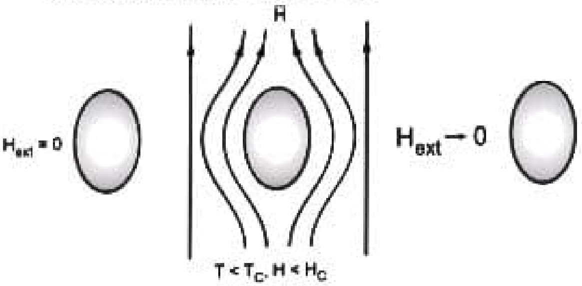
- Meissner and Ochsenfeld discovered in 1933 that a superconductor completely expels any magnetic field lines that were initially penetrating it in its normal state. This property is independent of the path by which the superconducting state is reached.

Path 1

- The sample is in superconducting state and is brought to the magnetic field. It is found that the magnetic flux is totally expelled from the sample.

Path 2

- The magnetic field is applied first to the sample in the normal state. Then the material be cooled to below T_c in the presence of the magnetic field. Meissner and Ochsenfeld found that the magnetic flux is totally expelled from the sample as it becomes superconducting. This expulsion of magnetic flux during the transition from normal to superconducting state is called as **Meissner Effect**.

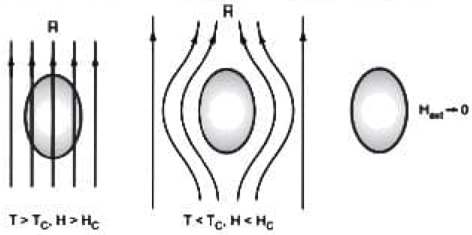


(a) Superconductor is initially outside the magnetic field

(b) When the superconductor is transported into the magnetic field, it pushes the field lines aside

(c) When the magnetic field is switched off

Fig. 5.12



(a) Superconductor in a magnetic field at T > T$_c$, magnetic field penetrates the body in normal state

(b) On cooling the superconductor in the presence of magnetic field, it expels the flux lines within the specimen at T = T$_c$. This is Meissner effect

(c) When the field is switched off, field is not trapped within the superconductor. Superconductivity is thus characterized by perfect diamagnetism

Fig. 5.13

Explanation of Meissner Effect

- When a superconducting sample is placed in a magnetic field, it induces currents which circulate on the surface of the specimen in a manner that it creates a magnetic field everywhere equal and opposite to the applied magnetic field.

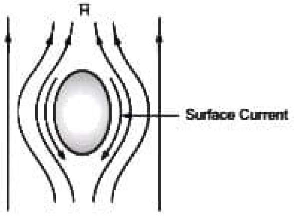
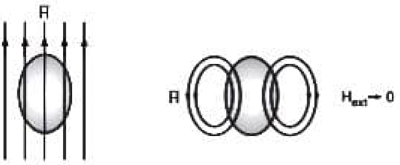


Fig. 5.14 : Meissner effect

- Meissner effect cannot be explained by the assumption that a superconductor is a resistanceless conductor. A superconductor is not just a perfect conductor but has an additional property. A material in the superconducting state does not permit any magnetic flux to exist within the body of the material.
- When a perfect conductor is cooled in a magnetic field until its resistance becomes zero, the magnetic field in the material is frozen or trapped in the material. It cannot change subsequently, irrespective of the applied field. Therefore, a conductor does not exhibit diamagnetic behaviour even slightly.



(a) Conductor held in magnetic field is cooled to the state of zero electrical resistance

(b) When magnetic field H$_{ext}$ is switched off, magnetic field is trapped in the ideal conductor

Fig. 5.15

- The magnetic induction inside the specimen is given by,

$$B = \mu_0 (H + M) \quad \text{(Normal state } T > T_c\text{)}$$

where, H – external applied field

M – magnetisation produced within the specimen

For $T < T_c$, $B = 0$

$$\therefore \mu_0 (H + M) = 0 \quad \text{(Superconducting state)}$$

$$\Rightarrow H = -M$$

The susceptibility of the material,

$$\chi = \frac{M}{H} = -1 \quad \text{(Perfect diamagnetism)}$$

- Thus, the superconducting state is characterized by perfect diamagnetism. Meissner effect conclusively proves whether a particular material has become a superconductor or not. Because of Meissner effect, superconducting materials strongly repel external magnets, it leads to both **Levitation Effect** and **Suspension Effect**.

5.12 TYPES OF SUPERCONDUCTORS

- There are two types of superconductors: type I and type II. There is no difference in the mechanism of superconductivity in both the types. Both have similar thermal properties at the transition temperature in zero magnetic field.
- The difference lies in their behaviour in a magnetic field, particularly in Meissner effect.

5.12.1 Type-I Superconductors

- In a type-I superconductor, the transition from a superconducting state to normal state, in the presence of a magnetic field, occurs sharply at the critical value H$_c$. At this point, the field penetrates completely.

- Below H_c, type-I superconductors are perfectly diamagnetic. They completely expel the magnetic field from the interior of the specimen. Upto the critical field strength, magnetization of the material grows in proportion to the external field. At the transition temperature, it suddenly drops to zero to the normal conducting state.
- The magnetic field penetrates only the surface layer and current flows only in this layer. Aluminum and lead are examples of type-I superconductors.
- As superconductivity gets destroyed at low values of critical field, type-I superconductors cannot be used in solenoids for producing large magnetic fields. Such superconductors are also called as **Soft Superconductors**.

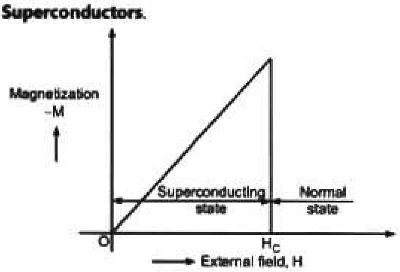


Fig. 5.16 : Magnetization curve for a type-I superconductor

5.12.2 Type-II Superconductors

- Type-II superconductor, also known as **Hard Superconductor** is characterized by two critical fields H_{c1} and H_{c2}. ($H_{c1} < H_c < H_{c2}$). It exists in three states: super-conducting, mixed and normal.

Superconducting State

- This occurs upto a critical field H_{c1}. The magnetization increases with the applied magnetic field and the external magnetic flux is completely expelled from the interior of the material.

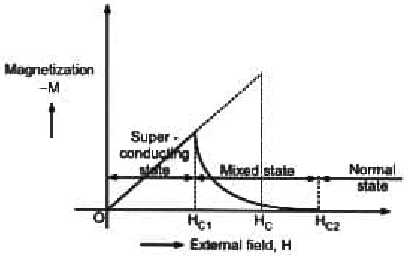


Fig. 5.17 : Magnetization curve in type-II superconductor

Mixed State

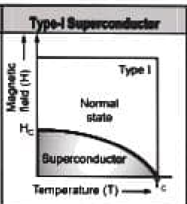
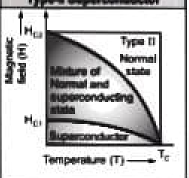
- This region extends from H_{c1} to H_{c2}. At H_{c1}, the magnetic flux penetrates the material. Between H_{c1} and H_{c2}, the material is in a mixed state magnetically but electrically it is a superconductor. Meissner effect is incomplete. In this region, the superconductor is threaded by flux lines and is said to be in a **Vortex State**. Value of H_{c2} may be 100 times higher than H_c (~20 to 50 Wb/m^2).
- As superconductivity is retained upto high values of magnetic fields, type-II superconductors are found useful in applications where high magnetic fields are created. Commercial solenoids wound with type-II superconductors produce high, steady magnetic fields above 10 T.
- Once the magnetic field is created by a superconductor solenoid, it does not require electrical power to maintain it. But the solenoid must be kept below critical transition temperature.

Normal State

- When the magnetic field exceeds critical field strength H_{c2}, magnetization vanishes completely. The sample is penetrated by the external field and superconductivity is destroyed. The specimen reverts from superconducting state to normal state.
- Type-II superconductors have a distinguishing feature. The supercurrents arising in an external magnetic field can flow not only on the surface but also in its bulk. The magnitude of the currents carried is also large when the magnetic field is between H_{c1} and H_{c2}.

Table 5.2: Types of Superconductor – Differences

[May 18]

Sr.	Property	Type-I Superconductor	Type-II Superconductor
1.	Variation of magnetic field with temperature		
2.	Critical magnetic field	Has one critical magnetic field H_c.	Has two critical magnetic fields H_{c1} and H_{c2}.
3.	Transition from superconducting to normal state	Transition from superconducting state to normal state in the presence of a magnetic field occurs sharply at the critical value H_c.	If external magnetic field is less than H_{c1}, material remains superconductor. When external magnetic field increases above H_{c2}, their superconductivity is destroyed.

Sr.	Property	Type-I Superconductor	Type-II Superconductor
4.	Magnetization below and above critical magnetic field	They are perfectly diamagnetic below H_c and completely expel magnetic field from interior of the superconducting phase.	For $H_{c1} < H < H_{c2}$ they exist in magnetically mixed and electronically superconducting state.
5.	Change in magnetization with external magnetic field	Upto H_c magnetization of the material grows in proportion to the external field and then abruptly drops to zero at the transition to the normally conducting state.	The magnetization of Type-II superconductors grows in proportion to the external field upto H_{c1}. The external magnetic flux is expelled from the interior of the material till then. At H_{c1}, magnetic field lines begin penetrating the material. As magnetic field increases further, the magnetic flux through the material increases. At H_{c2}, magnetization vanishes completely. External magnetic field penetrates completely and superconductivity is destroyed.
6.	Current carrying capacity	They are poor carriers of electrical current.	They are good carriers of electrical current.
7.	Magnetic field generation capacity	About 0.01 to 0.2 Wb/m² (value of H_c).	About 20 to 50 Wb/m² (Value of H_{c2}).
8.	Applications as magnets	Not much useful due to low H_c.	Useful due to high H_{c2}.
9.	Examples	Aluminium, lead, indium	Transition metals and alloys consisting of niobium, silicon and vanadium, Nb-Ti alloys, Nb_3Sn, etc.

5.13 APPLICATIONS OF SUPERCONDUCTIVITY

- The phenomenon of superconductivity finds numerous applications which can be broadly classified into two types.

1. **Large-Scale Applications**

   These are applications requiring large currents, long lengths of superconductors in environments where the magnetic field may be several tesla (1 tesla = 10^4 Oersted). Examples include magnets and power transmission lines, transformers and generators, where current densities of atleast 10^5 amps/cm² are required. Superconductors are more advantageous than normal conductors because of their lower resistance and hence smaller power loss.

2. **Small-Scale Applications**

These are applications involving minute amounts of current or fields. Examples are detection systems like SQUIDS.

5.14 LARGE-SCALE APPLICATIONS

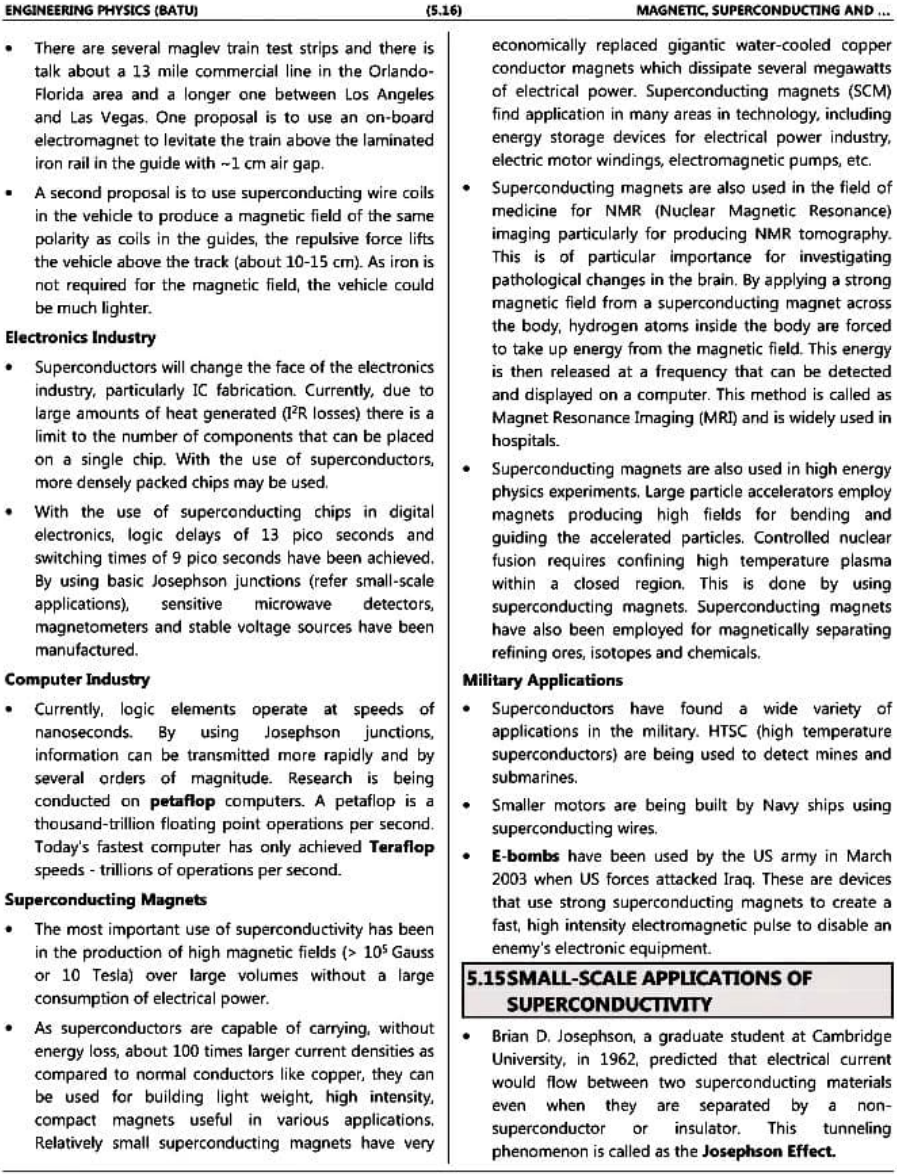
- The cost of energy consumption in the world and the electrical energy in particular are staggering. It is said that about one-fifth the power generated is lost due to I^2R losses. The elimination of even a small fraction of the resistive load will have a staggering impact.
- Another important area of application is the use of high temperature superconductors in the production of strong magnetic fields above the 2 Tesla level. This will eliminate the use of iron cores in motors, generators and transformers resulting in reduced size, weight and losses from iron cores.

Wires and Superconducting Magnets

- As R = 0 for a superconductor, there are no I^2R losses. There is no energy dissipation associated with the flow of a current through a superconductor. A current set up in a closed loop of a superconductor persists, almost forever, without decay.
- Superconducting wires could be used for very economical long distance power transmission, as energy dissipation is low and electrical power transmission can be done at a lower voltage level. Electric generators made with superconducting wire are more efficient than conventional generators wound with copper wire.

Magnetic levitation (Maglev)

- The zero magnetic induction in a superconductor is responsible for levitation effects.
- This phenomenon has led to one of the most spectacular applications, maglev or magnetically levitated train. Superconducting magnetic coils produce the magnetic repulsion required to levitate the train. Maglev trains will not slide over the rails but will float on an air cushion over a magnetised track. As there is no mechanical friction, speeds upto 500 km/hr can be achieved easily. As these trains are capable of very high speeds, they can compete with short hop plane flights in crowded air corridors.



- There are several maglev train test strips and there is talk about a 13 mile commercial line in the Orlando-Florida area and a longer one between Los Angeles and Las Vegas. One proposal is to use an on-board electromagnet to levitate the train above the laminated iron rail in the guide with ~1 cm air gap.
- A second proposal is to use superconducting wire coils in the vehicle to produce a magnetic field of the same polarity as coils in the guides, the repulsive force lifts the vehicle above the track (about 10-15 cm). As iron is not required for the magnetic field, the vehicle could be much lighter.

Electronics Industry

- Superconductors will change the face of the electronics industry, particularly IC fabrication. Currently, due to large amounts of heat generated (I^2R losses) there is a limit to the number of components that can be placed on a single chip. With the use of superconductors, more densely packed chips may be used.
- With the use of superconducting chips in digital electronics, logic delays of 13 pico seconds and switching times of 9 pico seconds have been achieved. By using basic Josephson junctions (refer small-scale applications), sensitive microwave detectors, magnetometers and stable voltage sources have been manufactured.

Computer Industry

- Currently, logic elements operate at speeds of nanoseconds. By using Josephson junctions, information can be transmitted more rapidly and by several orders of magnitude. Research is being conducted on **petaflop** computers. A petaflop is a thousand-trillion floating point operations per second. Today's fastest computer has only achieved **Teraflop** speeds - trillions of operations per second.

Superconducting Magnets

- The most important use of superconductivity has been in the production of high magnetic fields (> 10^5 Gauss or 10 Tesla) over large volumes without a large consumption of electrical power.
- As superconductors are capable of carrying, without energy loss, about 100 times larger current densities as compared to normal conductors like copper, they can be used for building light weight, high intensity, compact magnets useful in various applications. Relatively small superconducting magnets have very economically replaced gigantic water-cooled copper conductor magnets which dissipate several megawatts of electrical power. Superconducting magnets (SCM) find application in many areas in technology, including energy storage devices for electrical power industry, electric motor windings, electromagnetic pumps, etc.
- Superconducting magnets are also used in the field of medicine for NMR (Nuclear Magnetic Resonance) imaging particularly for producing NMR tomography. This is of particular importance for investigating pathological changes in the brain. By applying a strong magnetic field from a superconducting magnet across the body, hydrogen atoms inside the body are forced to take up energy from the magnetic field. This energy is then released at a frequency that can be detected and displayed on a computer. This method is called as Magnet Resonance Imaging (MRI) and is widely used in hospitals.
- Superconducting magnets are also used in high energy physics experiments. Large particle accelerators employ magnets producing high fields for bending and guiding the accelerated particles. Controlled nuclear fusion requires confining high temperature plasma within a closed region. This is done by using superconducting magnets. Superconducting magnets have also been employed for magnetically separating refining ores, isotopes and chemicals.

Military Applications

- Superconductors have found a wide variety of applications in the military. HTSC (high temperature superconductors) are being used to detect mines and submarines.
- Smaller motors are being built by Navy ships using superconducting wires.
- **E-bombs** have been used by the US army in March 2003 when US forces attacked Iraq. These are devices that use strong superconducting magnets to create a fast, high intensity electromagnetic pulse to disable an enemy's electronic equipment.

5.15 SMALL-SCALE APPLICATIONS OF SUPERCONDUCTIVITY

- Brian D. Josephson, a graduate student at Cambridge University, in 1962, predicted that electrical current would flow between two superconducting materials even when they are separated by a non-superconductor or insulator. This tunneling phenomenon is called as the **Josephson Effect.**

- It has been applied to electronic devices such as the SQUID, an instrument capable of detecting and measuring extremely weak magnetic fields.

5.15.1 Josephson Effect

Josephson Junction

Two superconductors connected by a thin layer of insulating material (~ 1-2 nm) is called a **Josephson Junction**. Under suitable conditions, Josephson found that remarkable effects were associated with the tunneling of superconducting electron pairs from a superconductor, through a layer of an insulator, into another superconductor. This junction is called a **Weak Link**. The effect found to be associated with the pair tunneling is called **Josephson Effect**.

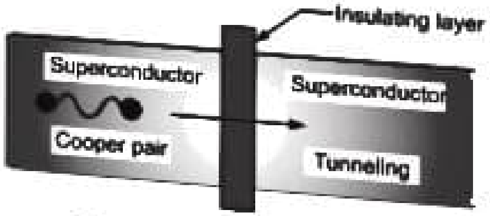


Fig. 5.18 : Josephson junction

(I) DC Josephson Effect

- When two superconductors are separated by a thin insulating layer, Cooper pairs tunnel through the junction and current flows across the junction without any external applied voltage. If this current does not exceed critical current I_c, voltage across the junction is zero. This effect is

- In such a case, the energies of the Cooper pair on both the sides of the barrier differ by 2 eV. The alternating supercurrents are accompanied by the emission or absorption of electromagnetic radiation.

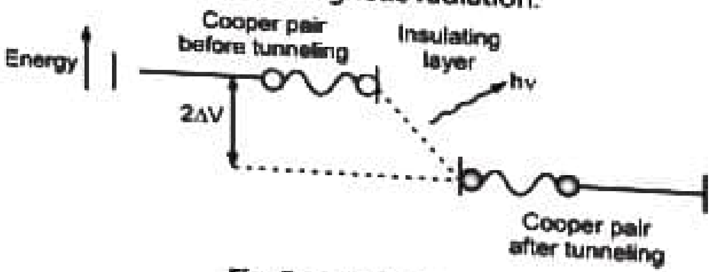


Fig. 5.20 : AC effect

- If ΔV is the finite potential difference between the superconductors, the electron pairs on opposite sides of the barrier differ in energy by an amount $2\Delta V = 2$ eV.
- Hence, frequency υ of the associated photon will be given by,

$$h\upsilon = 2\,eV \quad \text{or} \quad \upsilon = 2\left(\frac{e}{h}\right)V.$$

- Josephson suggested the determination of h/e from this relation after measuring applied voltage and frequency of emitted radiation. This experiment was carried out between 1967 and 1968. It is one of the simplest methods available to measure the fundamental constant.

5.16 INTRODU

- For a better knowledge of semiconductors, one should understand the properties of semiconductors on the basis of band theory of solids. For this, elementary knowledge of electronic configuration of atoms and quantum numbers is quite essential.

5.16.1 Electron Energy States of an Isolated Atom

- An isolated atom of an element with atomic number Z and mass number A consists of a positively charged nucleus, with Z protons and (A – Z) neutrons around which Z electrons revolve in different orbitals. The orbits are characterized by a set of four quantum numbers n, l, m_l and m_s.
- The distribution of electrons in an atom i.e., energy states decide the properties of the element to which the atom belongs. The energy of an electron in an atom depends on n as well as l i.e. energy of the electron is a function of n, l or $E = E(n, l)$.
- As n and l can have only discrete values, the energy E will have discrete values. The energy states characterized by n, l numbers are generally degenerate i.e. electrons with different set of quantum numbers will have the same energy, due to different m_l values for a given l. The state with same n and l will be $(2l + 1)$ degenerate.
- The number of electrons that can have the same energy E (n, l) with given n and l is $2(2l + 1)$, the factor 2 is due to two possible values of m_s for each m_l. The state s is non-degenerate and has two electrons. But p, d, f states are respectively 3-fold, 5-fold, 7-fold degenerate and the number of electrons in those states are 6, 10, 14 respectively.
- As such the energy states of an isolated atom will be quite discrete. The energy states of an isolated lithium atom are shown in Fig. 5.21.

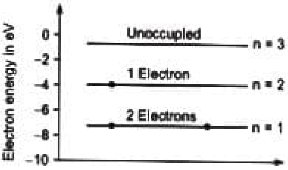


Fig. 5.21

5.17 BAND THEORY OF SOLIDS

- A solid is an aggregate of atoms in very close proximity. For example, a crystal is a periodic arrangement of atoms in which the structure is built up by a regular repetition of a small unit called a **Unit Cell**.
- The energy states of an isolated atom consist of discrete energy levels. But when the atoms are brought into close proximity as in a crystal, the outermost or valence electrons of adjacent atoms interact with each other. The inner or non-valence electrons do not interact significantly at any realizable interatomic distance because they are too closely associated with the nuclei.
- As per Pauli's exclusion principle, since not more than two interacting electrons may have the same energy level, new levels must be established which are discrete but only infinitesimally different. The separation between split energy sublevel is of the order of 10^{-23} eV. This group of related levels in a polyatomic material is called an **Energy Band**.
- In short, in crystals or solids, the allowed energy levels of an atom are modified by the proximity of other atoms in such a way that the discrete energy levels of the individual atoms become bands in solids.
- Each band contains as many discrete levels as there are atoms in the material. In a solid containing N atoms, there are N possible energy levels in each band such that, only two electrons of opposite spin may occupy the same energy level. Thus, the N levels will accommodate a maximum of 2N electrons. In other words, a band formed from N atoms contains 2N energy states.

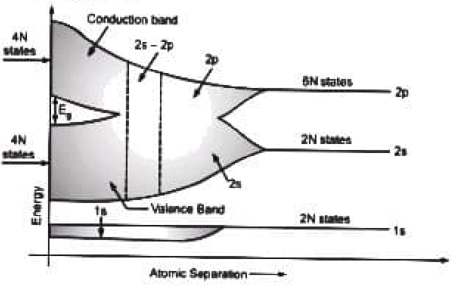


Fig. 5.22 : Formation of energy bands in a diamond crystal

- The imaginary formation of a diamond crystal from isolated carbon atoms is shown in Fig. 5.22. Each isolated carbon atom has an electron structure $1s^2\ 2s^2\ 2p^2$. Each atom has available two 1s states, two 2s states and six 2p states and higher states.
- If we consider N atoms, there will be 2N states of 1s type, 2N states of 2s type and 6N states of 2p type. As the interatomic spacing decreases, these energy levels

split into bands beginning with the outer (n = 2) shell. As the 2s and 2p bands grow, they merge into a single band composed of a mixture of energy levels.

- This band of '2s–2p' levels contains 8N available states. As the distance between atoms approaches the equilibrium interatomic spacing of diamond, this band splits into two bands separated by an energy gap or band gap E_g. The upper band is known as the **Conduction Band** while the lower one is known as the **Valence Band**. Thus, the conduction band contains 4N states and the valence band also contains 4N states.
- So, apart from the low lying and tightly bound 1s levels, the diamond crystal has two bands of available energy levels separated by energy gap E_g. The energy gap E_g does not contain allowed energy levels for electrons to occupy. This gap is also called as **Forbidden Band**.
- The lower 1s band is filled with 2N electrons which originally resided in the collective 1s states of the isolated atoms. However, there were 4N electrons in the original isolated n = 2 shell. (2N in 2s states and 2N in 2p states). These 4N electrons must occupy states in the valence band or the conduction band in the crystal.
- At 0 K, the electrons will occupy the lowest energy states available to them. In the case of the diamond crystal, there are exactly 4N states in the valence band available to the 4N electrons. So at 0 K every state in the valence band will be filled while the conduction band will be completely empty of electrons.
- This arrangement of completely filled and empty energy bands has an important effect on the electrical conductivity of the material. As conduction band is completely empty, the diamond will serve as an insulator.

5.17.1 Valence Band, Conduction Band and Forbidden Energy Gap

Energy Band

- In solids or crystals, allowed energy levels are modified by the proximity of other atoms in such a way that discrete energy levels of individual atoms are converted into series of energy levels. The difference in the energy sublevels is of the order of 10^{-23} eV. This series of energy levels is called **Energy Band**.

Valence Band

- The electrons in the inner shells are strongly bonded to their nuclei while the electrons in the outermost shells are not strongly bonded to their nuclei. It is these electrons which are most affected, when a number of atoms are brought very close together during the formation of a solid. The electrons in the outermost shell are called **Valence Electrons**. The band formed by a series of energy levels containing the valence electrons is known as **Valence Band**.
- The valence band may be defined as a band which is occupied by valence electrons or highest occupied energy band. The valence band is completely filled with electrons at 0 K.

Conduction Band

- The next higher permitted energy band is called the **Conduction Band**. This band may be either empty or partially filled with electrons. Conduction band may be defined as the lowest unfilled permitted energy band. It lies just above the valence band.
- The electrons occupying conduction band are known as **Conduction Electrons** and these electrons move freely in the conduction band.

Forbidden Gap

- The conduction band and valence band are separated by a region or a gap known as **Forbidden Band** or **Forbidden Gap**. This band is collectively formed by a series of nonpermitted energy levels above the top of the valence band to the bottom of the conduction band and is a measure of E_g.
- Thus, E_g is the amount of energy that should be imparted to the electron in the valence band for its migration to the conduction band. These bands are shown in Fig. 5.23.

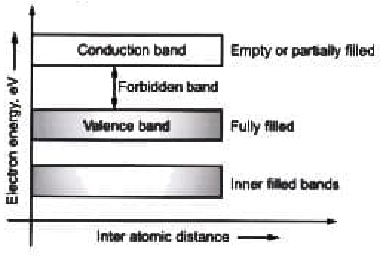


Fig. 5.23 : Valence band, conduction band and forbidden gap at T = 0 K

- If a valence electron happens to absorb enough energy, it jumps across the forbidden energy gap and enters the conduction band. Also, if a conduction electron happens to radiate too much energy, it will suddenly reappear in the valence band once again.

5.18 FERMI ENERGY

(a) Fermi Level in Conductors or Metals

- The statement that a solid is composed of N atoms implies that each atomic level splits into N-energy levels and bands of energy are formed. The filling of the bands follows a simple rule. States of lowest energy are filled first, then the next lowest and so on, till all the electrons are accommodated.
- The highest filled state is called the **Fermi level** and its corresponding energy is called the **Fermi Energy** E_F. The magnitude of E_F depends on the number of electrons per unit volume in the solid because the electron density determines how many electrons must go into the bands.
- At 0 K, all states upto E_F are full and all states above E_F are empty.
- At higher temperatures, the random thermal energy will empty a few states below E_F by elevating a few electrons to yet higher energy states. No transitions to states below E_F occur as they are full. Thus, an electron cannot change its state unless enough energy is provided to take it above E_F.
- The highest filled state in the highest energy band which contains electrons in a metal, at 0 K, is called the Fermi level and its corresponding energy is called the Fermi energy E_F.

(b) Fermi Level in Semiconductors

- In semiconductors, the Fermi level is a reference level that gives the probability of occupancy of states in conduction band as well as in valence band.
- In case of intrinsic semiconductors, the band picture consists of a band of completely filled states called as the **Valence Band** separated from a band of unoccupied states called as the **Conduction Band**, by an energy gap E_g. For an intrinsic semiconductor, the Fermi level lies at the centre of the forbidden band, indicating that the states occupied in conduction band are equal to the states unoccupied in valence band. In other words, for every electron in the conduction band, there is a hole in the valence band.
- So Fermi level in the semiconductors may be defined as the energy which corresponds to the centre of gravity of conduction electrons and holes when **Weighted** according to their energies.

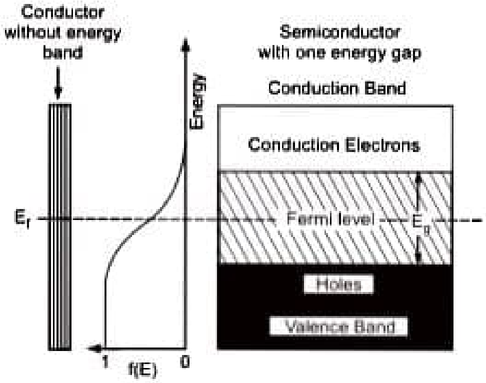


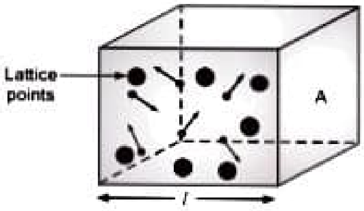
Fig. 5.24

- However, it is to be noted that Fermi level is only an abstraction. A hollow body can have a centre of gravity at the centre where there is no matter. Similarly, a material can have a Fermi level at an energy which is forbidden to all electrons. For example, in an intrinsic semiconductor, the Fermi level is at the centre of the forbidden band.

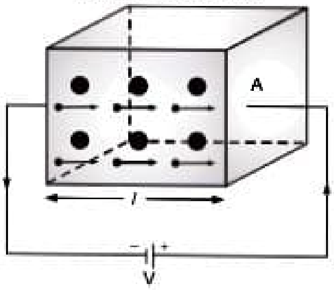
5.19 CONDUCTIVITY OF SEMICONDUCTORS

5.19.1 Conductivity of Conductors

According to the free electron model of an atom, the valence electrons are not attached to individual atoms. They move about freely along all directions among the atoms. These free electrons are called as conduction electrons and they form the **Free Electron Cloud** or **Free Electron Gas** or **Fermi Gas**.



(a) Random motion



(b) Directed motion

Fig. 5.25 : Current flow in conductors

In the absence of an external electrical field, the electrons move randomly in all directions [See Fig. 5.25 (a)]. When an electric field is applied to the metal, the random motion becomes directed [See Fig. 5.25 (b)]. This type of directed motion is known as **Drift**. The drift velocity v of the electrons depends upon the electron mobility μ_e and the applied electric field E.

- The drift velocity v is given by

$$v = \mu_e E \qquad \text{... (5.21)}$$

Let A = conductor cross-section area

n = electron density (i.e. number of free electrons per unit volume of the conductor)

l = length of the conductor

V = voltage applied across the two ends of the conductor

E = electric field applied.

- Then the charge crossing the cross-section 'A' of the conductor in unit time is equal to n × (v × A) e. This rate of flow of charge constitutes the current.

i.e.
$$I = n v A e \qquad \text{... (5.22)}$$

Substituting for v from equation (5.21), we get

$$I = n \mu_e E A e \qquad \text{... (5.23)}$$

Now, substituting for $E = \frac{V}{l}$ in equation (5.23), we get

$$I = n \mu_e \frac{V}{l} A e \qquad \text{... (5.24)}$$

$$\therefore \quad \frac{V}{I} = \frac{l}{A} \cdot \frac{1}{n\mu_e e} \qquad \text{... (5.25)}$$

By Ohm's law, we have,

$$R = \frac{V}{I} \qquad \text{... (5.26)}$$

$$\therefore \quad R = \frac{l}{A} \cdot \frac{1}{n \mu_e e} \qquad \text{... (5.27)}$$

But
$$R = \rho \frac{l}{A} \qquad \text{... (5.28)}$$

where, ρ is the resistivity of the conductor. Comparing equations (5.27) and (5.28), we get,

$$\rho = \frac{1}{n \mu_e e} \qquad \text{... (5.29)}$$

The unit of ρ is ohm-m.

- Conductivity 'σ' is defined as the reciprocal of resistivity.

∴ Conductivity,

$$\sigma = \frac{1}{\rho} = n e \mu \text{ mho/m} \qquad \text{... (5.30)}$$

- Now, current density J is defined as the current flowing across the unit cross section.

From (5.23), we have,

$$J = \frac{I}{A} = n e \mu_e E \qquad \text{... (5.31)}$$

From (5.30) and (5.31), we have

$$J = \sigma E \quad \text{or} \quad \sigma = \frac{J}{E}$$

5.19.2 Conductivity in a Semiconductor [May 18]

Fig. 5.26 shows the total current flow in a semiconductor. This current is a sum of current flow due to electron flow and hole flow.

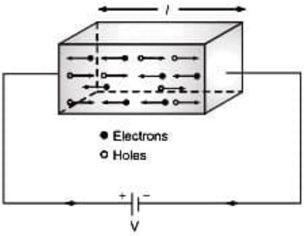


Fig. 5.26 : Current flow in a semiconductor

In a semiconductor, let

n_e = electron density in the conduction band

n_p = hole density in valence band

μ_e = electron mobility

μ_p = hole mobility

v_e = drift velocity of electrons

v_p = drift velocity of holes

A = cross section of the semiconductor

V = voltage applied across the semiconductor of length l

The current due to electrons is given by

$$I_e = n_e v_e A e \qquad \text{... (5.32)}$$

and the current due to holes is given by

$$I_p = n_p v_p Ae \qquad \dots (5.33)$$

Therefore, total current flowing through the semiconductor will be,

$$\text{Total current, } I = I_e + I_p$$

$$I = n_e v_e Ae + n_p v_p Ae$$

$$\therefore \quad I = Ae (n_e v_e + n_p v_p) \qquad \dots (5.34)$$

The drift velocity of a charged particle in electric field E is,

$$v = \mu E$$

$$\therefore \text{ For electrons, } v_e = \mu_e E$$

$$\text{and for holes, } v_p = \mu_p E$$

$$\text{But} \quad E = \frac{V}{l}$$

$$\therefore \quad v_e = \mu_e \frac{V}{l} \qquad \dots (5.35)$$

$$v_p = \mu_p \frac{V}{l} \qquad \dots (5.36)$$

Substituting equations (5.35) and (5.36) in equation (5.34), we get

$$I = Ae \left(n_e \mu_e \frac{V}{l} + n_p \mu_p \frac{V}{l} \right)$$

$$I = \frac{AeV}{l} (n_e \mu_e + n_p \mu_p) \qquad \dots (5.37)$$

$$\therefore \quad R = \frac{V}{I} = \frac{l}{Ae (n_e \mu_e + n_p \mu_p)} \qquad \dots (5.38)$$

$$\text{But} \quad R = \rho \frac{l}{A} \qquad \dots (5.39)$$

$\therefore$ Resistivity of the given semiconductor is given by [comparing equations (5.38) and (5.39)],

$$\rho = \frac{1}{e (n_e \mu_e + n_p \mu_p)} \text{ ohm-m} \qquad \dots (5.40)$$

The conductivity is reciprocal of resistivity.

$\therefore$ Conductivity

$$\sigma = \frac{1}{\rho} = e (n_e \mu_e + n_p \mu_p) \text{ mho/m} \qquad \dots (5.41)$$

Hence, conductivity in a semiconductor is a sum of conductivity due to both electrons and holes.

$$\text{Or} \quad \sigma_{sc} = \sigma_e + \sigma_p$$

From equation (5.34),

$$\frac{I}{A} = e (n_e \mu_e + n_p \mu_p) E$$

$\therefore$ The current density

$$J = \frac{I}{A} = e (n_e \mu_e + n_p \mu_p) E \qquad \dots (5.42)$$

From (5.41) and (5.42),

$$J = \sigma E$$

Case (i): Intrinsic Semiconductor

- For intrinsic semi conductors, number of electrons and holes are exactly same,

$$n_e = n_p = n_i$$

$\therefore$ Conductivity of an intrinsic semiconductor is

$$\sigma_i = e n_i (\mu_e + \mu_p)$$

Case (ii): N-type Extrinsic Semiconductor

- For N-type semiconductors, electron concentration is much greater than the hole concentration.

$$\therefore \quad n_e >> n_p \quad \text{or} \quad n_e \mu_e >> n_p \mu_p$$

$$\text{Hence} \quad \sigma_N \approx e n_e \mu_e$$

- If n_a is electron concentration or concentration of donor atoms, then,

$$\sigma_N \approx e n_d \mu_e \quad (\text{as } n_e \approx n_d)$$

Case (iii): P-type Extrinsic Semiconductor

- In P-type semiconductor, electron concentration is negligibly small in comparison to hole concentration.

$$\text{Then} \quad n_p >> n_e \quad \text{or} \quad n_p \mu_p >> n_e \mu_e$$

$$\therefore \quad \sigma_p = e n_p \mu_p$$

- If n_a is acceptor atom concentration then $\sigma_p \approx e n_a \mu_p$ (as $n_p \approx n_a$)

Problem 5.5: *Calculate the current produced in a small Germanium plate of area 1 cm² and of thickness 0.3 mm when a P.D. of 2 V is applied across the faces.*

Given:

$$n_i = 2 \times 10^{19} \text{ /m}^3$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\mu_e = 0.36 \text{ m}^2\text{/volt-sec}$$

$$\mu_h = 0.17 \text{ m}^2\text{/volt-sec}$$

Solution: Data:

$$A = 1 \times 10^{-4} \text{ m}^3$$

$$V = 2 \text{ volts}$$

$$l = 0.3 \text{ mm} = 0.3 \times 10^{-3} \text{ m}$$

Formula:

$$I = n_i e (\mu_e + \mu_h) \frac{V}{l} \cdot A$$

Solution:

$$I = 2 \times 10^{19} \times 1.6 \times 10^{-19} (0.36 + 0.17) \frac{2 \times 10^{-4}}{0.3 \times 10^{-3}}$$

$$= \boxed{1.13 \text{ amp.}}$$

Problem 5.6: *Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is 1.5 × 10^{16} / m^3 and the mobilities of electrons and holes are 0.12 and 0.05 m^2/V - sec respectively at room temperature.*

Data: $n_i = 1.5 \times 10^{16}/\text{m}^3$, $\mu_e = 0.12\ \text{m}^2/\text{V-sec}$,

$\mu_h = 0.05\ \text{m}^2/\text{V-sec}$.

Formula:

$$\sigma_{in} = \sigma_n + \sigma_p$$

$$\sigma_{in} = n_i\, e(\mu_e + \mu_h)$$

Solution:

$$\sigma_{in} = 1.5 \times 10^{16} \times 1.6 \times 10^{-19}\ (0.12 + 0.05)$$

$$= \boxed{4.1 \times 10^{-4}\ \text{mho/m}}$$

Problem 5.7: *Calculate the conductivity of the Germanium specimen if a donor impurity is added to the extent of one part in 10^8 Germanium atoms in room temperature.*

Given: *Avogadro number = 6.02 × 10^{23} atoms/moles*

At. wt. of Ge = 72.6

Density of Ge = 5.32 g/cm^3

Mobility μ_e = 3800 cm^2/ V-sec

Formula:

$$\sigma = e\, n_d\, \mu_e$$

Solution: Concentration of Ge atoms

$$= \frac{6.02 \times 10^{23}}{72.6} \times 5.32$$

$$= 4.41 \times 10^{22}\ /\text{cm}^3$$

Since there is one donor atom per 10^8 Germanium atoms then

$$n_d = \frac{4.41 \times 10^{22}}{10^8} = 4.41 \times 10^{14}\ /\ \text{cm}^3$$

In N-type semiconductor, n > p

$$\text{then } \sigma = e\, n_d\, \mu_e$$

$$= 1.6 \times 10^{-19} \times 4.41 \times 10^{14} \times 3800$$

$$= \boxed{0.268\ \text{mho/cm.}}$$

Problem 5.8: *The resistivity of an n-type semiconductor is 10^{-6} Ω cm. Calculate the number of donor atoms which must be added to obtain the resistivity.*

Given: μ_e = *1000 cm^2/V-sec.*

Data:

$$\rho = 10^{-6}\ \Omega\ \text{cm}$$

$$\mu_e = 1000\ \text{cm}^2/\text{V-sec}$$

Formula: Resistivity $\rho = \dfrac{1}{n_d\, e\, \mu_e}$

Solution:

$$n_d = \frac{1}{\rho\, e\, \mu_e}$$

$$\therefore \quad n_d = \frac{1}{10^{-6} \times 1.6 \times 10^{-19} \times 1000}$$

$$= \boxed{6.25 \times 10^{21}\ \text{atoms.}}$$

Problem 5.9: *Calculate the conductivity of extrinsic silicon at room temperature if the donor impurity added is 1 in 10^8 silicon atoms.*

Given: *At room temperature,*

$$n_i = 1.5 \times 10^{10}\ \text{per cm}^3$$

$$\mu_e = 1300\ \text{cm}^2\ /\ \text{volt-sec}$$

and number of silicon atoms per unit volume = 5 × 10^{22}.

Formula:

$$\sigma_n = n e\, \mu_e$$

Solution: If there is 1 donor atom per 10 silicon atoms, then the number of donor atoms per cm^3

$$n_d = \frac{\text{number of silicon atoms/unit volume}}{10^8}$$

$$= \frac{5 \times 10^{22}}{10^8} = 5 \times 10^{14}$$

Assuming all the donors are ionised and n >> p, hole conduction can be neglected.

$$\therefore \quad \sigma_n = n\, e\, \mu_e$$

$$\sigma_n = n_d\, e\, \mu_e$$

$$= 5 \times 10^{14} \times 1.6 \times 10^{-19} \times 1300$$

$$= \boxed{0.104\ \text{mho/cm.}}$$

Problem 5.10: *In Germanium, the energy gap is 0.75 eV. What is the wavelength at which Germanium starts to absorb light ?*

Data:

$$E_g = 0.75\ \text{eV}$$

Formula:

$$E_g = h\upsilon = \frac{hc}{\lambda}$$

Solution: Energy gap in a semiconductor is the minimum energy required to shift an electron from the top of valence band to the bottom of the conduction band. If photons of minimum energy hυ are absorbed by a material to enable electrons to cross the energy gap, then

$$h\upsilon = E_g$$

$$\therefore \quad E_g = h\upsilon = h\frac{c}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{\lambda}\ \text{J}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times \lambda}\ \text{eV}$$

$$\text{i.e.} \quad E_g = \frac{12400}{\lambda}\ \text{eV, if } \lambda \text{ is in A.}$$

$$\therefore \quad \lambda \equiv \frac{12400}{E_g} = \frac{12400}{0.75}$$

$$\therefore \quad \boxed{\lambda = 1653\ \text{A}^\circ}$$

Problem 5.11: *Calculate the average thermal velocity, the drift velocity and the mobility of electrons in copper in an electric field of 100 V/cm. Calculate also the density of the electric currents. The resistivity of copper is* 1.72×10^{-8} *ohm-m at 25°C. Boltzmann constant is* 1.38×10^{-23} *J/K, density of copper is* 8.9×10^{3} *kg/m³ and At. wt. is 63.54.*

Data: $E = 100$ V/cm, $\rho = 1.72 \times 10^{-8}\ \Omega\text{-m}$,

$k = 1.38 \times 10^{-23}$ J/K, density $= 8.9 \times 10^{3}$ kg/m³,

At. wt. = 63.54.

Formulae: (i) $v = \sqrt{\frac{3kT}{m}}$, (ii) $v_d = \mu E$, (iii) $\sigma = ne\mu = \frac{1}{\rho}$.

Solution: At equilibrium, the electrons follow the Maxwell-Boltzmann distribution. So their average K.E. for each degree of freedom is $\frac{1}{2}$ kT. For particles moving in three dimensions, we can write,

$$\frac{1}{2} mv^2 = \frac{3}{2} kT$$

$$\therefore \quad v = \left(\frac{3\,kT}{m}\right)^{1/2}$$

$$= \left(\frac{3 \times 1.38 \times 10^{-23} \times 298}{9.1 \times 10^{-31}}\right)^{1/2}$$

$$v = 1.16 \times 10^{5} \text{ m/sec.}$$

Since each copper atom contributes one valence electron to the conduction band, the number of electrons/m³ will be equal to the number of copper atoms/m³.

$$\therefore \text{No. of electrons/m}^3 = n = \frac{6.02 \times 10^{26} \times 8.9 \times 10^{3}}{63.54}$$

$$= 0.84 \times 10^{29} \text{ atoms/m}^3$$

$$\text{Mobility } \mu = \frac{1}{\rho \cdot n \cdot e}$$

$$\mu = \frac{1}{1.72 \times 10^{-8} \times 0.84 \times 10^{29} \times 1.6 \times 10^{-19}}$$

$$= 4.33 \times 10^{-3} \text{ m}^2\text{/volt-sec}$$

$$\text{Drift velocity } v_d = \mu \cdot E$$

$$= 4.33 \times 10^{-3} \times 100$$

$$\therefore \quad \boxed{v_d = 0.433 \text{ m/sec.}}$$

Problem 5.12: *Calculate the conductivity of pure silicon at room temperature when the concentration of carriers is* 1.6×10^{10} */cm³.*

$\mu_e = 1500$ *cm²/volt-sec*

$\mu_h = 500$ *cm²/volt-sec at room temperature*

Data:

$n_i = 1.6 \times 10^{10}/\text{cm}^3$

$\mu_e = 1500 \text{ cm}^3/\text{V-sec}$

$\mu_n = 500 \text{ cm}^3/\text{V-sec}$

Formula: $\sigma_{in} = \sigma_n + \sigma_p$

Solution:

$$\sigma_{in} = n_i\, e\, (\mu_e + \mu_n)$$

$$= 1.6 \times 10^{10} \times 1.6 \times 10^{-19}\,(1500 + 500)$$

$$= \boxed{5.12 \times 10^{-6} \text{ mho/cm}}$$

5.20 HALL EFFECT AND HALL COEFFICIENT

5.20.1 Hall Effect

[Dec. 17, 18, May 19]

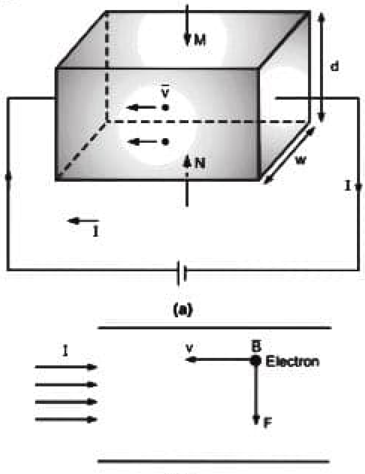
- It often becomes necessary to determine whether a material is an N-type or a P-type semiconductor. Measurement of conductivity alone does not give this information as no distinction can be made between hole and electron conduction.
- Hall effect is used to differentiate between the two types of carriers. It provides a means of determining the density and mobility of charge carriers and gives information about the sign of the predominant charge carrier.
- If a piece of conductor (metal or semiconductor) carrying a current is placed in a transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both the current and the magnetic field. This phenomenon is known as **Hall Effect** and the voltage so generated is called as **Hall Voltage**.

Explanation of the Effect

- Assume that the sample material is an N-type semiconductor. The current flow consists, almost entirely, of electrons moving from right to left. This movement corresponds to the direction of conventional current from left to right as shown in Fig. 5.27 (a).
- If v is the drift velocity of the electrons moving perpendicular to the magnetic field B, there is a downward force Bev acting on each electron. This causes the electrons to be deflected in the downward direction. This makes negative charges to accumulate on the bottom face of the slab [See Fig. 5.27 (b)] leaving positive ions on the top surface.
- This gives rise to a potential difference along the top and bottom faces of the specimen across points M and N with the bottom face being negative. This potential difference causes a field E_H in the negative y-direction

and so a force eE_H acts on the electrons in the upward direction.

- Under equilibrium, the upward force due to the electric field just balances the downward force due to the magnetic field.



(a)

(b)

Fig. 5.27 : Hall effect

Thus, $$e E_H = e B v$$

$$\therefore \quad E_H = v B \qquad \dots (5.43)$$

- If I is the current in the x-direction then,

$$I = n v A e$$

or $$v = \frac{I}{neA} \qquad \dots (5.44)$$

where n is the concentration of charge carriers.

$$\therefore \quad E_H = \frac{B I}{neA} \qquad \dots (5.45)$$

Also $$E_H = \frac{V_H}{d}$$

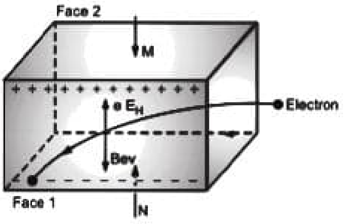


Fig. 5.28 : Motion of electrons in an n-type semiconductor

where V_H is the Hall voltage named after the scientist Hall who first predicted and measured the Hall voltage.

$$\therefore \quad V_H = E_H d \qquad \dots (5.46)$$

Substituting this in expression (5.45),

$$V_H = \frac{1}{ne} \cdot \frac{BId}{A} \qquad \dots (5.47)$$

or $$V_H = R_H \frac{BId}{A} \qquad \dots (5.48)$$

where $$R_H = \frac{1}{ne}$$

is the Hall coefficient for any charge e. ... (5.49)

- If J_x is the current density of charge carriers in x-direction then,

$$V_H = \frac{1}{ne} \cdot B J d \quad \left(\text{as } J = \frac{I}{A}\right) \qquad \dots (5.50)$$

In this specimen, as the dominant charge carriers are electrons,

$$\therefore \quad V_H = -\frac{1}{ne} B J d \qquad \dots (5.51)$$

- In expression (5.50), all three quantities V_H, B and J can be measured. Hence, Hall coefficient and current density can be found.

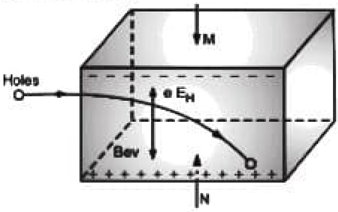


Fig. 5.29 : Motion of holes in p-type semiconductor

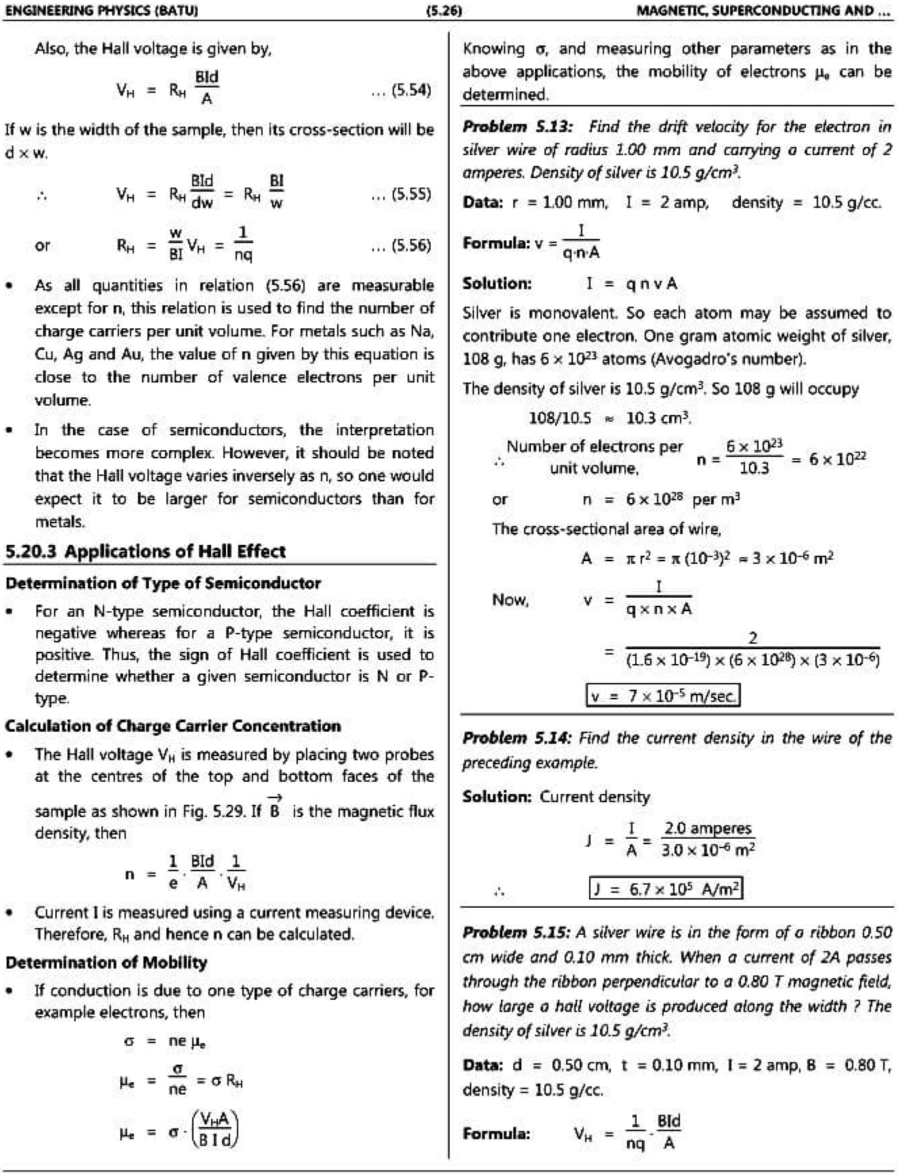
- Similarly, formulae can be derived for P-type semiconductors. All the formulae are same except that the Hall coefficient will be positive.
- The sign of the Hall voltage gives the sign of the charge carrier and this provides one of the few methods by which the sign of the charge carrier can be ascertained.

$$\therefore \text{ Hall voltage, } V_H = R_H \cdot \frac{BId}{A} = R_H B J d \qquad \dots (5.52)$$

5.20.2 Hall Coefficient (R_H) [Dec. 17, May 19]

- The Hall coefficient R_H is determined by measuring the Hall voltage that generates the Hall field. If V_H is the Hall voltage across the sample of thickness d then

$$V_H = E_H d \qquad \dots (5.53)$$



Also, the Hall voltage is given by,

$$V_H = R_H \frac{BId}{A} \quad \text{... (5.54)}$$

If w is the width of the sample, then its cross-section will be d × w.

$$\therefore \quad V_H = R_H \frac{BId}{dw} = R_H \frac{BI}{w} \quad \text{... (5.55)}$$

$$\text{or} \quad R_H = \frac{w}{BI} V_H = \frac{1}{nq} \quad \text{... (5.56)}$$

- As all quantities in relation (5.56) are measurable except for n, this relation is used to find the number of charge carriers per unit volume. For metals such as Na, Cu, Ag and Au, the value of n given by this equation is close to the number of valence electrons per unit volume.
- In the case of semiconductors, the interpretation becomes more complex. However, it should be noted that the Hall voltage varies inversely as n, so one would expect it to be larger for semiconductors than for metals.

5.20.3 Applications of Hall Effect

Determination of Type of Semiconductor

- For an N-type semiconductor, the Hall coefficient is negative whereas for a P-type semiconductor, it is positive. Thus, the sign of Hall coefficient is used to determine whether a given semiconductor is N or P-type.

Calculation of Charge Carrier Concentration

- The Hall voltage V_H is measured by placing two probes at the centres of the top and bottom faces of the sample as shown in Fig. 5.29. If $\vec{B}$ is the magnetic flux density, then

$$n = \frac{1}{e} \cdot \frac{BId}{A} \cdot \frac{1}{V_H}$$

- Current I is measured using a current measuring device. Therefore, R_H and hence n can be calculated.

Determination of Mobility

- If conduction is due to one type of charge carriers, for example electrons, then

$$\sigma = ne\,\mu_e$$

$$\mu_e = \frac{\sigma}{ne} = \sigma R_H$$

$$\mu_e = \sigma \cdot \left(\frac{V_H A}{B\,I\,d}\right)$$

Knowing σ, and measuring other parameters as in the above applications, the mobility of electrons μ_e can be determined.

Problem 5.13: *Find the drift velocity for the electron in silver wire of radius 1.00 mm and carrying a current of 2 amperes. Density of silver is 10.5 g/cm³.*

Data: r = 1.00 mm, I = 2 amp, density = 10.5 g/cc.

Formula: $v = \frac{I}{q \cdot n \cdot A}$

Solution: $I = q\,n\,v\,A$

Silver is monovalent. So each atom may be assumed to contribute one electron. One gram atomic weight of silver, 108 g, has 6×10^{23} atoms (Avogadro's number).

The density of silver is 10.5 g/cm³. So 108 g will occupy

$$108/10.5 \approx 10.3\ \text{cm}^3.$$

$$\therefore \quad \text{Number of electrons per unit volume,} \quad n = \frac{6 \times 10^{23}}{10.3} = 6 \times 10^{22}$$

$$\text{or} \quad n = 6 \times 10^{28}\ \text{per m}^3$$

The cross-sectional area of wire,

$$A = \pi r^2 = \pi (10^{-3})^2 \approx 3 \times 10^{-6}\ \text{m}^2$$

$$\text{Now,} \quad v = \frac{I}{q \times n \times A}$$

$$= \frac{2}{(1.6 \times 10^{-19}) \times (6 \times 10^{28}) \times (3 \times 10^{-6})}$$

$$\boxed{v = 7 \times 10^{-5}\ \text{m/sec.}}$$

Problem 5.14: *Find the current density in the wire of the preceding example.*

Solution: Current density

$$J = \frac{I}{A} = \frac{2.0\ \text{amperes}}{3.0 \times 10^{-6}\ \text{m}^2}$$

$$\therefore \quad \boxed{J = 6.7 \times 10^{5}\ \text{A/m}^2}$$

Problem 5.15: *A silver wire is in the form of a ribbon 0.50 cm wide and 0.10 mm thick. When a current of 2A passes through the ribbon perpendicular to a 0.80 T magnetic field, how large a hall voltage is produced along the width ? The density of silver is 10.5 g/cm³.*

Data: d = 0.50 cm, t = 0.10 mm, I = 2 amp, B = 0.80 T, density = 10.5 g/cc.

Formula: $V_H = \frac{1}{nq} \cdot \frac{BId}{A}$

Solution: The atomic weight of silver is 108, so the number of atoms in 1 cm^3 is

$$n = (6 \times 10^{23}) \left(\frac{10.5}{108}\right) \approx 6 \times 10^{22} \text{ per cm}^3$$

Silver is monovalent and we can assume that each atom contributes one electron.

$\therefore$ Number of electrons per m^3 = 6×10^{28}

$$A = 0.05 \times 0.001 = 5 \times 10^{-5} \text{ m}^2$$

$$\text{Hall voltage, } V_H = \frac{1}{nq} \cdot \frac{BI \cdot d}{A}$$

$$= \frac{1}{6 \times 10^{28} \times 1.6 \times 10^{-19}} \times \frac{0.80 \times 2.0 \times 0.05}{5 \times 10^{-5}}$$

$$\approx \boxed{1.67 \times 10^{-7} \text{ volt.}}$$

Problem 5.16: *A copper specimen having length 1 metre, width 1 cm and thickness 1 mm is conducting 1 amp current along its length and is applied with a magnetic field of 1 Tesla along its thickness. It experiences a Hall effect and a Hall voltage of 0.074 microvolts appears along its width. Calculate the Hall coefficient and the mobility of electrons in copper. Conductivity of copper is* $\sigma = 5.8 \times 10^7 \, (\Omega m)^{-1}$.

Data: $l = 1$ m, $d = 1$ cm $= 10^{-2}$ m, $t = 1$ mm $= 10^{-3}$ m,

$B = 1$ Tesla, I = amp. $V_H = 0.074 \times 10^{-6}$ volts,

$\sigma = 5.8 \times 10^7 \, (\Omega m)^{-1}$

Formulae: (i) $V_H = R_H \cdot \frac{BId}{A}$, (ii) $\sigma = \frac{\mu}{R_H}$.

Solution: (i)

$$R_H = \frac{V_H \cdot A}{BId} = \frac{0.074 \times 10^{-6} \times 10^{-2}}{1 \times 1 \times 10^{-2}}$$

$$= \boxed{0.074 \times 10^{-6} \text{ m}^3/\text{coulomb}}$$

(ii)

$$\mu = \sigma R_H = 5.8 \times 10^7 \times 0.074 \times 10^{-6}$$

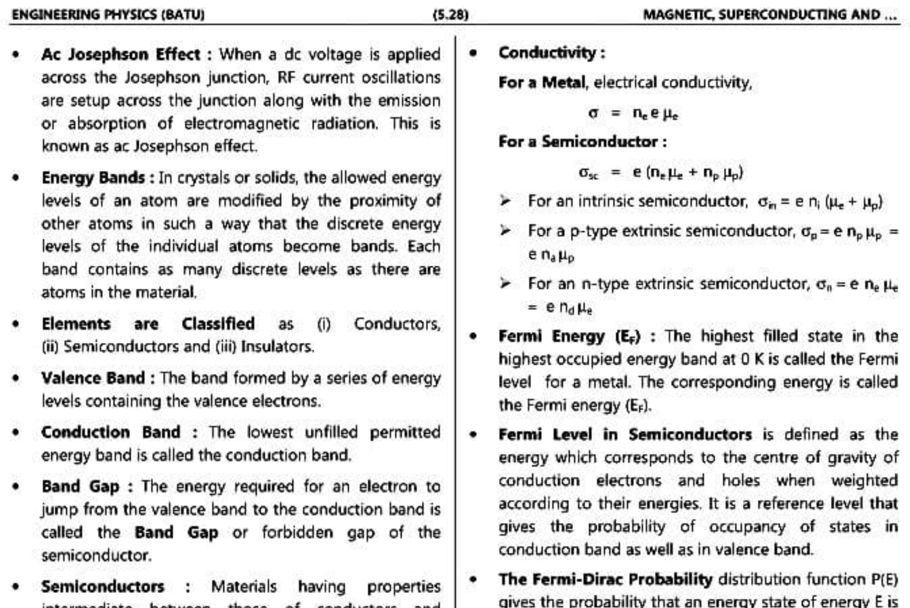
$$= \boxed{4.292 \text{ m}^2/\text{volt-sec.}}$$

SUMMARY

- **Dipole :** The torque experienced by a magnet when placed in an external field is called dipole moment.
- **Types of Magnetic Materials :** Depending upon the permanent magnetic dipole present a magnetic material can be classified as (i) Paramagnetic (ii) Ferromagnetic (iii) Antiferromagnetic and (iv) Ferromagnetic.
- **Hysterics Curve :** A plot of B-H of a magnetic material is called the hysterics curve.
- **Free Electrons :** In conductors the electrons are free to move within the specimen.
- **Free Electron Theory :** Proposed by Drude-Lorentz treats electrons like gas molecule.
- **Quantized Energy :** Quantum-mechanical treatment of free electron gave the concept of discrete energy level.
- **Conductivity :** The electrical conductivity is given by $\sigma = ne\mu$.
- **Ohm's Law :** Ohm's law is given by $J = 6E$
- **Superconductivity:** A conductor having zero electrical resistance is called a superconductor and this phenomenon is called as superconductivity.
- **Critical Transition Temperature (T_c) :** The temperature below which superconductivity is exhibited.
- **Superconductivity Vanishes :** If temperature, magnetic field and current density exceed the critical value. For superconducting state, $T < T_c$, $H < H_c$ and $J < J_c$.
- **Meissner Effect :** The expulsion of magnetic field/flux from the interior of the specimen, when cooled below the critical temperature.
- Variation of critical magnetic field with temperature is given by

$$H_c(T) = H_c(0) \left[1 - \frac{T}{T_c}\right]^2$$

  where $H_c(0)$ is the critical magnetic field at 0 K.
- **Type-I Superconductors :** Are pure specimens which expel completely magnetic field lines. They exhibit perfect diamagnetism. They are also called as soft superconductors.
- **Type-II Superconductors :** Are characterized by two critical fields. Between the two critical fields, the magnetic flux partially penetrates the material. Above the upper critical field flux, penetration is total. They are also called as hard superconductors.
- **Josephson Effect :** Tunneling of current between two superconductors separated by an insulator is known as Josephson effect.
- **Dc Josephson Effect :** The flow of a dc current across the Josephson junction, in the absence of any electric or magnetic field is known as dc Josephson effect.



- **Ac Josephson Effect :** When a dc voltage is applied across the Josephson junction, RF current oscillations are setup across the junction along with the emission or absorption of electromagnetic radiation. This is known as ac Josephson effect.
- **Energy Bands :** In crystals or solids, the allowed energy levels of an atom are modified by the proximity of other atoms in such a way that the discrete energy levels of the individual atoms become bands. Each band contains as many discrete levels as there are atoms in the material.
- **Elements are Classified** as (i) Conductors, (ii) Semiconductors and (iii) Insulators.
- **Valence Band :** The band formed by a series of energy levels containing the valence electrons.
- **Conduction Band :** The lowest unfilled permitted energy band is called the conduction band.
- **Band Gap :** The energy required for an electron to jump from the valence band to the conduction band is called the **Band Gap** or forbidden gap of the semiconductor.
- **Semiconductors :** Materials having properties intermediate between those of conductors and insulators.
- **Semiconductors are of Two Types :** (i) Intrinsic and (ii) Extrinsic.
- **Intrinsic Semiconductors** are those which are pure (free from electroactive and crystalline defects).
- **Doping** is the process of adding an impurity to intrinsic semiconductors to increase its conductivity.
- **Extrinsic Semiconductors** are obtained by doping an intrinsic semiconductor. They are of two types: (i) p-type extrinsic semiconductor, (ii) n-type extrinsic semiconductor.
- **P-Type Semiconductor :** An extrinsic semiconductor formed by doping a trivalent impurity is called a p-type semiconductor. In this type, holes are the majority charge carriers and electrons are the minority charge carriers.
- **N-Type Semiconductor :** An extrinsic semiconductor formed by doping a pentavalent impurity is called as n-type semiconductor. In this type, electrons are the majority charge carriers and holes are the minority charge carriers.
- **Conductivity :**

  For a Metal, electrical conductivity,

  $$\sigma = n_e e \mu_e$$

  For a Semiconductor :

  $$\sigma_{sc} = e (n_e \mu_e + n_p \mu_p)$$

  - For an intrinsic semiconductor, $\sigma_{in} = e\, n_i (\mu_e + \mu_p)$
  - For a p-type extrinsic semiconductor, $\sigma_p = e\, n_p \mu_p = e\, n_a \mu_p$
  - For an n-type extrinsic semiconductor, $\sigma_n = e\, n_e \mu_e = e\, n_d \mu_e$
- **Fermi Energy (E_F) :** The highest filled state in the highest occupied energy band at 0 K is called the Fermi level for a metal. The corresponding energy is called the Fermi energy (E_F).
- **Fermi Level in Semiconductors** is defined as the energy which corresponds to the centre of gravity of conduction electrons and holes when weighted according to their energies. It is a reference level that gives the probability of occupancy of states in conduction band as well as in valence band.
- **The Fermi-Dirac Probability** distribution function P(E) gives the probability that an energy state of energy E is occupied by an electron at T K.

  $$P(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$
- **The Fermi Level** in intrinsic semiconductors is exactly in the middle of the forbidden gap.

  $$E_F = \frac{E_c + E_v}{2}$$
- The position of the Fermi level in a **P-Type Extrinsic Semiconductor** is close to the valence band as holes are the majority charge carriers.
- The Fermi level in an **N-Type Extrinsic Semiconductor** is close to the conduction band as electrons are the majority charge carriers.
- **Hall Effect :** When a current carrying specimen (I) is placed in a transverse magnetic field (B), an electric field 'E' is induced in the specimen perpendicular to both I and B. This phenomenon is called as Hall effect and the voltage hence developed is called as Hall voltage.
- **Hall Voltage,** $V_H = R_H \cdot \frac{BId}{A}$
- **Hall Coefficient,** $R_H = \frac{1}{nq}$.

IMPORTANT FORMULAE

- Drift velocity, $\bar{v}_x = \frac{eE}{m}\tau$
- Collision time, $\tau = \frac{\lambda}{v_x}$
- Mobility of electron, $\mu = \frac{\bar{v}_x}{E}$
- Conductivity, $\sigma = \frac{n e^2 \tau}{m}$
- $\sigma = ne\mu$
- The magnetic induction inside the specimen (superconductor) is given by,

$$B = \mu_0 (H + M) \qquad \text{(Normal state } T > T_c\text{)}$$

For $T < T_c$, $B = 0$

$\therefore \mu_0 (H + M) = 0$ (Superconducting state)

or $H = -M$

- The variation of critical field with temperature is,

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)\right]^2$$

- The critical current density,

$$J_c = \frac{\text{Critical current}}{\text{Area of the ring}}$$

$$J_c = \frac{2\pi R H_c}{\pi R^2} = \frac{2H_c}{R}$$

- Ohm's law is given by $J = 6E$
- Drift velocity, $v = \mu_e E$
- For metals
  - Current, $I = n \mu_e \frac{V}{l} \cdot A \cdot l$
  - Resistivity, $\rho = \frac{1}{n e \mu_e}$
  - Conductivity, $\sigma = \frac{1}{\rho} = n \mu_e$
- For semiconductors
  - Current, $I = l \cdot A \cdot \frac{V}{\rho} (n \mu_e + p \mu_h)$ (for semiconductors)
  - Resistivity, $\rho = \frac{1}{e (n \mu_e + p \mu_h)}$ (for semiconductors)
  - Conductivity, $\sigma = \frac{1}{\rho} = e (n \mu_e + p \cdot \mu_h)$
  - Conductivity, $\sigma_i = e n_i (\mu_e + \mu_h)$ (for intrinsic semiconductors)
  - Conductivity, $\sigma_n = n_e e \mu_e = n_d e \mu_e$ (for n-type semiconductors)
  - Conductivity, $\sigma_p = n_h e \mu_h = n_a e \mu_h$ (for p-type semiconductors)
- Hall voltage, $V_H = R_H \cdot \frac{BId}{A}$
- Hall coefficient, $R_H = \frac{1}{nq}$
- Mobility, $\mu = \sigma R_H$.

UNSOLVED PROBLEMS

1. A material has resistivity of 1.54×10^{-8} Ωm at room temperature. There are $5.8 \times 10^{28}/m^3$ electrons. The Fermi energy of the conductor is 5.5 eV. Calculate (i) the velocity of electrons with Fermi energy, (ii) mean free path. [**Ans.** 1.39×10^6 m/s, 5.56×10^{-8} m]
2. The relaxation time of conduction electrons in a material is 3×10^{-14} s. If the density of electrons is $5.8 \times 10^{28} / m^3$, calculate the resistivity of the material and the mobility of electrons. [**Ans.** 2.017×10^{-28} Ωm, $5.31 \times 10^{-3}\ m^2 V^{-1} s^{-1}$]
3. A metal has resistivity 1.43×10^{-8} Ωm and density of conduction electrons $6.5 \times 10^{28} / m^3$. Find relaxation time. [**Ans.** 3.82×10^{-14} sec]
4. The mobilities of carriers in intrinsic germanium sample at room temperature are $\mu_e = 3600$ cm²/volt-sec and $\mu_p = 1700$ cm²/volt-sec. If the density of electrons is same as holes and is equal to 2.5×10^{13} per cm³, calculate the conductivity. [**Ans.** 2.12 mho/m]
5. Calculate the number of acceptors to be added to a Germanium sample to obtain the resistivity $\rho = 10$ ohm. cm. Given $\mu = 1700$ cm²/volt-sec. [**Ans.** 3.676×10^{14} per cm³]
6. At room temperature the conductivity of a silicon crystal is 5×10^{-4} mho/cm. If the electron and hole mobilities are 0.14 m²/volt-sec and 0.05 m²/volt-sec, determine the density of carriers. [**Ans.** $1.64 \times 10^{16} / m^3$]
7. The specific density of tungsten is 18.8 g/cm³ and its atomic weight is 184.0. Assume that there are two free electrons per atom. Calculate the concentration of free electrons. Avogadro No. = 6.025×10^{23} / gmole. [**Ans.** $2.5 \times 10^{23}/cm^3$]
8. Compute the conductivity of copper for which $\mu_e = 34.8$ cm²/volt-sec and d = 8.9 gm/cm³. Assume that there is one free electron per atom. Av. No. = 6.025×10^{23} / g mole, atomic weight of Cu = 63.5.
   If an electric field is applied across such a copper bar with an intensity of 10 V/cm, find the average velocity of free electrons. [**Ans.** 47.02×10^{-4} mho/cm, 348 cm/sec.]

9. The resistivity of copper wire of diameter 1.03 mm is 6.51 ohm per 300 m. The concentration of free electrons in copper is $8.4 \times 10^{28} / m^3$. If the current is 2 A, find (a) mobility, (b) drift velocity, (c) conductivity.
[Ans. 0.413 m^2/volt-sec, 0.286×10^{-20} m/sec, 55.5×10^8 mho/m]
10. Calculate the energy gap in silicon if it is given that it is transparent to radiation of wavelength greater than 11000 A°. [Ans. 1.13 eV]
11. An N-type semiconductor is to have a resistivity of 10 ohm-cm. Calculate the number of donor atoms which must be added to achieve this.
Assume, $\mu_e = 500$ cm^2/volt-sec. [Ans. 12.5×10^{23}]

EXERCISE

1. What is magnetic material? Give the types of magnetic material?
2. Explain the hystersis curve. Classify magnetic materials on the basis of hytersis curve.
3. Discuss the Drude-Lorentz classical free electron theory.
4. Derive the formula for the electrical conductivity.
5. Derive the relation between mobility and conductivity.
6. What are the assumptions of classical free electron theory ? Derive expression of conductivity of metals.
7. Write short notes on (a) Relaxation time, (b) Mean free path, (c) Collision time, (d) Drift velocity, (e) Mobility.
8. State an explain microscopic ohm's law. Derive formula for it.
9. Explain the effect of temperature on conductivity of a material.
10. What is superconductivity ? What are the characteristics of superconductors ?
11. Explain Meissner effect, isotope effect, critical temperature and critical field.
12. What are the types of superconductors ? Where do they find application ?
13. Enumerate the different applications of superconductors. How are they advantageous as compared to normal conductors ?
14. Explain some properties of type-I and type-II superconductors.
15. Describe in brief the formation of energy bands in solids.
16. Explain the terms : valence band, conduction band and forbidden energy gap.
17. Derive an expression for conductivity in an intrinsic and extrinsic semiconductor.
18. Explain Hall effect and Hall coefficient.
19. What is Fermi energy ? Show the location of Fermi energy levels in intrinsic and extrinsic semiconductors.
20. State Hall effect. Derive the formula for Hall voltage and Hall coefficient.
21. What is Fermi function ? Show that the Fermi level lies at the centre of the energy gap in an intrinsic semiconductor.
22. State and explain the applications of Hall effect.

UNIVERSITY QUESTIONS

December 2017

1. On the basis of domain theory explain B-H curve and hence explain retentively and coactivity. [6]
2. What is Superconductivity ? Explain Meissner Effect in Superconductors. [2+4]
3. What is Hall effect? Derive an expression for Hall Coefficient. [6]

May 2018

1. Discuss the different types of magnetic materials interms of magnetic moments. [6]
2. Prove Bohr magneton μ_B = eh/ 2m. Differentiate between hard and soft magnetic materials. [6]
3. What is Microscopic Ohm's Law? Differentiate between Type I and Type II superconductors. [6]
4. Derive an expression for conductivity in an intrinsic and extrinsic semiconductor. [6]
Calculate conductivity of pure silicon when the concentration of carriers is 1.6×10^{10} $/cm^3$, and $\mu e = 1500$ cm^2/V-s, $\mu h = 500$ cm^2/V-s.

December 2018

1. What are Ferrites and Garnets? Write their general formula. Determine the magnetization and flux density of the diamagnetic, if its magnetic susceptibility is -0.4×10^{-5} and magnetic field in it is 10^4 A/m. [6]
2. Prove Bohr Magnetron μ_B = eh/2m. Differentiate between hard and soft magnetic materials. [6]
3. What is Superconductivity? Explain Meissner effect in superconductor. [6]
4. What is Hall effect? Derive an expression for Hall coefficient of p and n type semiconductor. [6]

May 2019

1. Write formula of Ferrites and Garnets. [6]
2. Explain Meissner effect in superconductors.
3. What is Hall Effect? Derive an expression for Hall voltage VH and Hall coefficient R_H. [6]

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